MATHEMATICS

GRADE 8

STUDENT TEXT BOOK

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UNIT 1
RATIONAL NUMBERS

Learning Outcomes: At the end of this unit, learners will able to:

- Define and represent rational numbers as fractions
- Show the relationship among ℕ, W, ℤ and ℚ .
- Order rational numbers.
- Solve problems involving addition, Subtraction, Multiplication and division of rational numbers
- Apply Rational Numbers to solve practical problems.
- Aware the four operations as they relate to Rational Numbers.

Main Contents

1.1 The concept of Rational numbers
1.2 Comparing and Ordering Rational numbers
1.3 Operation and Properties of Rational numbers
1.4 Application of Rational numbers
Summary
Review Exercise

INTRODUCTION

Many times throughout your mathematics lessons, you will be manipulating specific kinds of numbers that are related to your real life activities. So, it is important to understand how mathematicians classify numbers and what kinds of major classifications exist.

In the previous grades you have learnt about the set of Natural numbers, Whole numbers and Integers and their basic properties. In this unit you will learn about the set of numbers which contains the other set of numbers (i.e. $\mathbb{N}$, W, $\mathbb{Z}$) called Rational numbers. And also you will learn about the basic properties, operations and real life applications of rational numbers.

1.1 The concept of Rational numbers

Competencies: At the end of this sub-topic, students should:

- ❖ Describe the concept of Rational Numbers practically.
- ❖ Express Rational Numbers as fractions.

Group work 1.1

Describe the following questions with your groups

1. Provide an example of each of the following numbers
   a. Natural numbers smaller than 10.
   b. Whole number that is not a natural number
   c. Integers that is not a whole number
2. Which of the following set of numbers include the other set of numbers?
   a. Whole numbers
   b. Integers
   c. Natural numbers
3. Define a rational number in your own words.
4. Solomon has 3 cats and 2 dogs. He wants to buy a toy for each of his pets. Solomon has 22 Birr to spend on pet toys. How much can he spend on each pet? Write your answer as a fraction and as an amount in Birr and Cents.

1.1.1. Representation of Rational Numbers on a Number line

Competency: At the end this sub-topic, students should:

- Represent rational numbers as a set of fractions on a number line.

Revision on Fractions

A fraction represents the portion or part of the whole thing. For example, one-half, three-quarters. A fraction has two parts, namely numerator (the number on the top) and denominator (the number on the bottom).

Example 1.1:

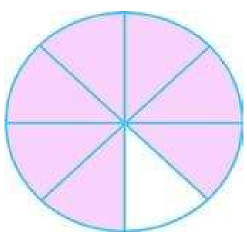


Figure 1.1 DVD

The shaded part is $\frac{7}{8}$ of the DVD

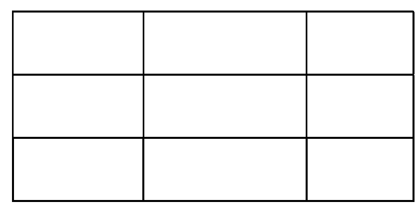


Figure 1.2 Rectangular fields

one part is one-ninth, $\frac{1}{9}$ of the rectangular field

Example 1.2:

If three-fifths of the green area be covered by indigenous plants, then find the numerator and denominator of the covered area.

Figure 1.3 Green area

Solution:

$\frac{3}{5}$

Numerator

Denominator

In grade 6 and 7 you have discussed the important ideas about fractions and integers.

i. **Proper fraction:** A fraction in which the numerator is less than the denominator.

ii. **Improper fraction:** A fraction in which the numerator is greater than or equal to the denominator.

If an improper fraction is expressed as a whole number and proper fraction, then it is called mixed fraction.

Integers are represented on number line as shown below in figure 1.4.

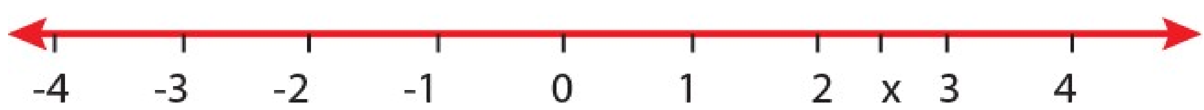


Figure 1.4

What number is represented by the marked letter x on the number line above? You observe that the number x is greater than 2 but less than 3. So, it belongs to the interval between 2 and 3. Thus x is not a natural number, or a whole number, or an integer. What type of a number is?

Using the above discussion, we define a rational number as follow:

Definition 1.1

A number that can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$, is called a rational number.

Example 1.3:

$\frac{1}{5}, \frac{3}{7}, \frac{3}{5}, -\frac{8}{2}, -\frac{14}{4}$ and $\frac{5}{9}$ are rational numbers.

Note: The set of rational numbers is denoted by $\mathbb{Q}$.

How can we locate rational numbers on number line?

Rational number can be represented on a number line by considering the following facts.

I. Positive rational numbers are always represented on the right side of zero and negative rational numbers are always represented on the left side of zero on a number line.

II. Positive proper fractions always exist between zero and one on number line.

III. Improper fractions are represented on number line by first converting into mixed fraction and then represented on the number line.

Example 1.4

Sketch a number line and mark the location of each rational numbers.

a. $\frac{2}{5}$ b. $\frac{3}{2}$ c. $-\frac{3}{4}$ d. $-\frac{5}{2}$

Solution:

a) Since $\frac{2}{5} > 0$, and proper, so it lies on the right side of 0 and on the left side of 1. How can we locate?

Divide the number line between 0 and 1 into 5 equal parts. Then the second part of the fifth parts will be a representation of $\frac{2}{5}$ on number line.

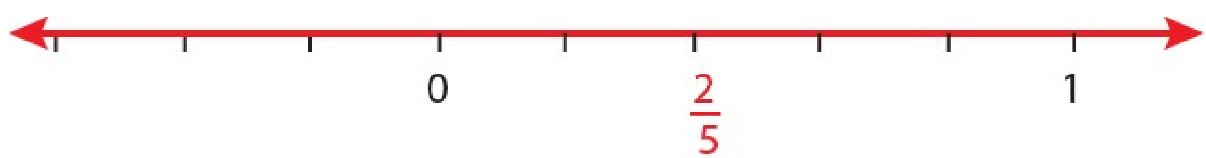


Figure 1. 5

b) Since $\frac{3}{2}$ is an improper fraction, first convert to mixed fraction to find between which whole numbers the fraction exists on the number line.

Thus, $\frac{3}{2} = 1\frac{1}{2}$. The fraction lies between 1 and 2 at $\frac{1}{2}$ point. Now, divide the number line between 1 and 2 in two equal parts and then the 1st part of 2 parts will be the required rational number on the number line.

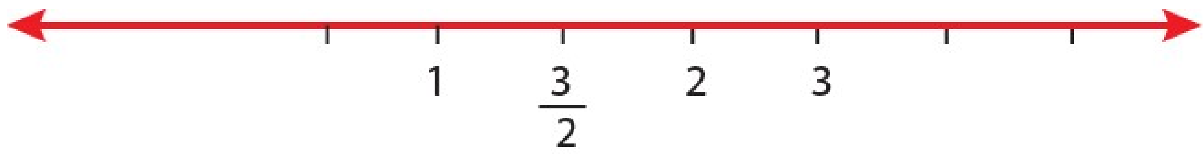


Figure 1.6

c) Since $-1 < -\frac{3}{4} < 0$, the fraction will lie between -1 and 0.To represent on the number line, divide the number line between -1 and 0 in to 4 equal parts and the third part of the four parts will be $-\frac{3}{4}$.

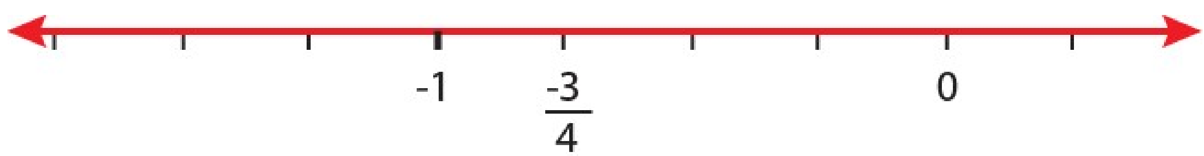


Figure 1.7

d) Since $-\frac{5}{2} < 0$ and improper, first change in to mixed fraction. That is, $-\frac{5}{2} = -2\frac{1}{2}$. To represent on the number line divide the number line between -3 and -2 in to two equal parts and the first part of the two parts is $-\frac{5}{2}$.

Note:

Two rational numbers are said to be opposite, if they have the same distance from 0 but in different sides of 0.

For instance $\frac{3}{2}$ and $-\frac{3}{2}$ are opposites.

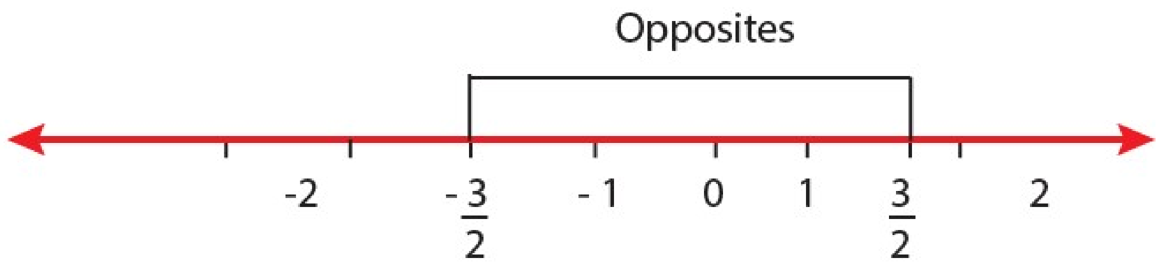


Figure 1.8

Exercise 1.1

1. Consider the following number line.

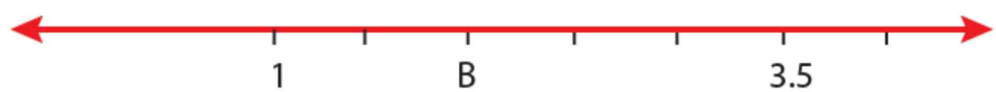


Figure 1.10

Select a reasonable value for point B.

a) 0.5 b) 3.6 c) – 0.2 d) 2

2. Between what consecutive integers the following rational numbers exist?

   a) $\frac{3}{7}$ b) $\frac{8}{5}$ c) $-\frac{3}{5}$ d) $-\frac{9}{5}$

3. Change the following improper fractions to mixed fractions.

   a) $\frac{32}{5}$ b) $-\frac{27}{10}$ c) $\frac{7}{3}$

4. Represent the following rational numbers on a number line.

   a) $\frac{5}{6}$ b) $\frac{3}{5}$ c) $-\frac{5}{6}$ d) $-\frac{8}{5}$ e) $2\frac{2}{5}$

5. If you plot the point -8.85 on a number line, would you place it to the left or right of -8.8? Explain.

6. Find the opposite of the following rational numbers.

   a) $\frac{4}{5}$ b) $-\frac{2}{3}$ c) $2\frac{3}{5}$ d) $-3\frac{4}{7}$

1.1.2. Relationship among W, ℤ and ℚ

Competency: At the end of this section, students should:

- Describe the relationship among the sets ℕ, W, ℤ and ℚ.

In the previous grades of mathematics lesson you have learnt about the sets of natural numbers (ℕ), whole numbers (W) and integers (ℤ).In this subsection you will discuss the relationship among these set of numbers with the other set of number which is rational numbers.

Recall that:

- ✓ A collection of items is called a set.
- ✓ The items in a set are called elements and is denoted by ∈.
- ✓ A Venn diagram uses intersecting circles to show relationships among sets of numbers.

The Venn diagram below shows how the set of natural numbers, whole numbers, integers, and rational numbers are related to each other.

When a set is contained within a larger set in a Venn diagram, the numbers in the smaller set are members of the larger set. When we classify a number, we can use Venn diagram to help figure out which other sets, if any, it belongs to.

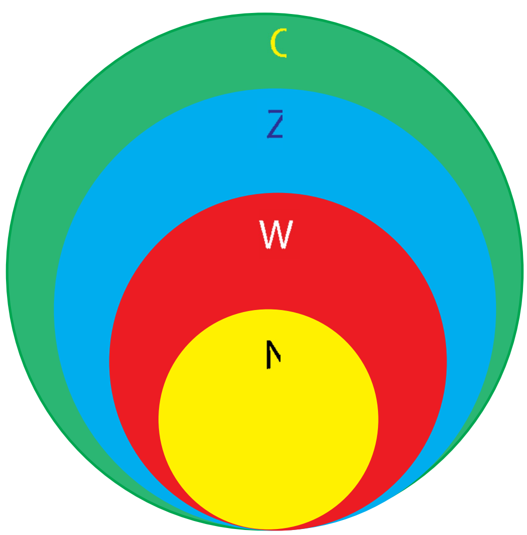


Figure 1.11. Venn diagram

Example 1.5:

Classify the following numbers by naming the set or sets to which it belongs.

a. -13 b. $\frac{1}{7}$ c. $-\frac{5}{76}$ d. 10

Solution:

a. integer, rational number

b. rational number

c. rational number

d. natural number, whole number, integer, rational number.

Example 1.6:

Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain.

Solution: No, because a whole number is an integer.

Exercise 1.2

1. Solomon says the number 0 belongs only to the set of rational numbers. Explain his error.
2. Write true if the statement is correct and false if it is not.
   a) The set of numbers consisting of whole numbers and its opposites is called integers.
   b) Every natural number is a whole number.
   c) The number $-3\frac{2}{7}$ belongs to negative integers.

1.1.3. Absolute value of Rational numbers

Competency: At the end of this section, students should:

- Determine the absolute value of a rational number.

Activity 1.1

1. What is the distance between 0 and 5 on the number line? Between 0 and -5 on the number line?

The absolute value of a rational number describes the distance from zero that a number is on a number line without considering direction.

For example, the absolute value of a number is 5 means the point is 5 units from zero on the number line.

Definition 1.2: The absolute value of a rational number 'x', denoted by $|x|$, is defined as:

$$|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$$

Example 1.7:

a. $|6| = 6$

b. $|0| = 0$

c. $|-15| = -(-15) = 15$

Example 1.8:

Simplify each of the following absolute value expressions.

a. $|8-3|$ b. $|-25+13|$ c. $|0-10|$

Solution:

a. Since $8-3 = 5$ and $5 > 0$, we have $|8-3| = |5| = 5$

b. Since $-25+13 = -12$ and $-12 < 0$, we have

$|-25+13| = |-12| = -(-12) = 12$

c. Since $0-10 = -10$ and $-10 < 0$, we have

$|0-10| = |-10| = -(-10) = 10$

Equation involving absolute value

Definition 1.3: An equation of the form $|x| = a$ for any rational number a is called an absolute value equation.

Geometrically the equation $|x| = 8$ means that the point with coordinate x is 8 units from 0 on the number line. Obviously the number line contains two points that are 8 units from the origin, one to the right and the other to the left of the origin. Thus $|x| = 8$ has two solutions $x = 8$ and $x = -8$.

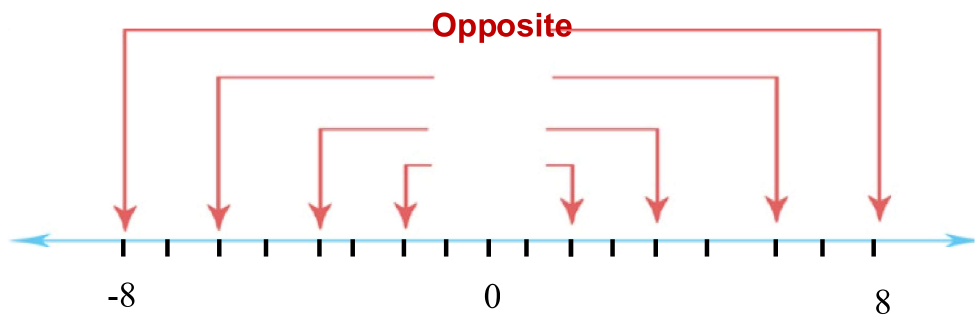


Figure 1.12

Note

The solution of the equation $|x| = a$ for any rational number a, has

i. Two solutions $x = a$ and $x = -a$ if $a > 0$.

ii. One solution, $x = 0$ if $a = 0$ and

iii. No solution, if $a < 0$.

Example 1.9:

Solve the following absolute value equations.

a. $|x| = 13$ b. $|x| = 0$ c. $|x| = -6$

Solution:

a. $|x| = 13$

Since, $13 > 0$, $|x| = 13$ has two solutions:

$x = 13$ and $x = -13$

b. $|x| = 0$

If $|x| = 0$, then $x = 0$

c. $|x| = -6$

Since, $-6 < 0$, $|x| = -6$ has no solution.

Exercise 1.3

1. Complete the following table.

x	-7	$\frac{3}{5}$			$-\frac{2}{3}$
$\lvert x \rvert$			0	$2\frac{5}{9}$	

2. Find all rational numbers whose absolute values are given below

a. 3.5 b. $\frac{4}{7}$ c. $\frac{11}{6}$ d. $3\frac{2}{5}$

3. Evaluate each of the following expressions.

a. $|-5|+|5|$

b. $|-13|-|-8|+|7|$

c. $|0|+2\frac{1}{3}$

d. $|-8+5|$

e. $|-\frac{1}{13}|+|-15|-15$

4. Evaluate each of the following expressions for the given values of x and y.

a. $5x - |x - 3|$, $x = -5$

b. $|x|-x+9$, $x = 3$

c. $|x+y|-|x|$, $x = -3$ and $y = 6$

d. $|x|+|y|$, $x = 5$ and $y = -10$

e. $-3|x+6|$, $x = -5$

f. $\frac{|x|-|5y|}{|x+y|}$, $x = 4$ and $y = 8$

5. Solve the following absolute value equations.

a. $|x| = 8$ b. $|x| = \frac{3}{5}$

Challenge problem

6. Solve the following absolute value equations.

a. $|x+4| = 10$

b. $4|x+3| = 123$

c. $3-2|x-5| = 9$

d. $|5x-3| = \frac{5}{2}$

e. $|x-5| = 3\frac{2}{3}$

1.2. Comparing and Ordering Rational numbers

Competency: At the end of this sub-topic students should:

- Compare and order Rational numbers.

1.2.1. Comparing Rational numbers

In day to day activity, there are problems where rational numbers have to be compared. For instance, win and loss in games; positive and negative in temperature; profit and loss in trading etc.

Activity 1.2

Insert <, = or > to express the corresponding relationship between the following pairs of numbers.

a. 0 ------- 15

b. -3 ------- -5

c. 6.7 ------- 6.89

d. $\frac{12}{3}$ ------- $\frac{8}{2}$

Comparing Decimals

A rational number $\frac{a}{b}$ can be expressed as a decimal number by dividing the numerator a by the denominator b.

Note: Decimal numbers are compared in the same way as comparing other numbers: By comparing the different place values from left to right. That is, compare the integer part first and if they are equal, compare the digits in the tenths place, hundredths place and so on.

Example 1.10:

Compare the following decimal numbers.

a. 4.25-----12.33

b. 15.52-----15.05

c. 45.667 ----- 45.684

Solution:

a. since 4 <12, then 4.25 < 12.33

b. the integer part of the two numbers are equal, then move to the tenth place and compare: 5> 0, then 15.52 > 15.05

c. here the corresponding integer part and the tenth place numbers are equal, so we move to the hundredth place: $6 < 8$. Thus $45.667 < 45.684$

Comparing Fractions

Comparing fractions with the same denominator

If the denominators of two rational numbers are the same, then the number with the greater numerator is the greater number. That is

$\frac{a}{b}$ and $\frac{c}{b}$ are a given rational numbers and $\frac{a}{b} > \frac{c}{b}$ if and only if $a > c$.

Example 1.11:

a. $\frac{15}{7} > \frac{13}{7}$ because 15 >13

b. $\frac{10}{3} < \frac{15}{3}$ because 10 < 15

Fractions that represent the same point on a number line are called Equivalent fractions. For any fraction $\frac{a}{b}$ and m is a rational number different from 0 ($m \neq 0$), then $\frac{a}{b} = \frac{a}{b} \times \frac{m}{m}$.

Comparing fractions with different denominators

In order to compare any two rational numbers with different denominators, you can use either of the following two methods:

Method 1:

Change the fractions to equivalent fractions with the same denominators.

Step 1. Determine the LCM of the positive denominators.

Step 2. Write down the given rational numbers with the same denominators.

Step 3. Compare the numerators of the obtained rational numbers.

Example 1.12:

Compare the following pairs of rational numbers.

a. $\frac{3}{5}$ and $\frac{1}{2}$ b. $\frac{11}{16}$ and $\frac{7}{8}$

Solution:

a. To compare $\frac{3}{5}$ and $\frac{1}{2}$

i. Find the LCM of 5 and 2 which is 10.

ii. Express the rational numbers with the same denominator 10.

$\frac{3}{5} = \frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$ and $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$

iii. Since $6 > 5$, $\frac{6}{10} > \frac{5}{10}$.

Therefore, $\frac{3}{5} > \frac{1}{2}$.

b. To compare $\frac{11}{16}$ and $\frac{7}{8}$

i. Find the LCM of 16 and 8 which is 16.

ii. Express the rational numbers with the same denominator 16.

$\frac{11}{16} = \frac{11}{16} \times \frac{1}{1} = \frac{11}{16}$ and $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$

Since $11 < 14$, $\frac{11}{16} < \frac{14}{16}$

Therefore, $\frac{11}{16} < \frac{7}{8}$

Method 2. (ross- product method)

Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers with positive denominators. Then

I. $\frac{a}{b} < \frac{c}{d}$, if and only if $ad < bc$

II. $\frac{a}{b} > \frac{c}{d}$, if and only if $ad > bc$

III. $\frac{a}{b} = \frac{c}{d}$, if and only if $ad = bc$

Example 1.13:

a. $\frac{5}{7} < \frac{8}{3}$ because $5 \times 3 = 15 < 7 \times 8 = 56$

b. $-\frac{12}{9} < \frac{6}{7}$ because $-12 \times 7 = -84 < 9 \times 6 = 54$

c. $\frac{9}{5} > \frac{11}{7}$ because $9 \times 7 = 63 > 5 \times 11 = 55$.

Comparing Rational numbers using number line

Note:

For any two different rational numbers whose corresponding points are marked on the number line, then the one located to the left is smaller.

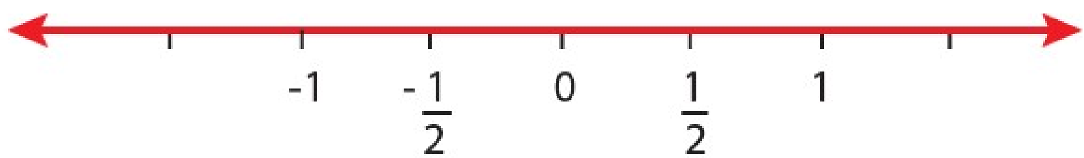


Figure 1.13

Thus, $-1 < -\frac{1}{2}$, $-\frac{1}{2} < 0$, $0 < \frac{1}{2}$

From the above fact, it follows that:

- Every positive rational number is greater than zero.
- Every negative rational number is less than zero.
- Every positive rational number is always greater than every negative rational number.
- Among two negative rational numbers, the one with the largest absolute value is smaller than the other. For instance, $-45 < -23$ because $|-45| > |-23|$.

Exercise 1.4

1. Which of the following statements are true and which are false?

   a. $-0.15 < 1.5$ b. $2\frac{3}{5} > 3\frac{2}{5}$ c. $\left|-\frac{3}{5}\right| < \frac{3}{5}$ d. $\frac{12}{8} = \frac{10}{15}$

   e. $3\frac{5}{7} > \frac{21}{6}$ f. $\frac{5}{12} > \frac{7}{18}$ g. $6.53 < 6.053$ h. $3\frac{4}{7} = \frac{25}{7}$

2. Insert (>, = o r <) to express the corresponding relationship between the following pairs of numbers.

a. $\frac{15}{9}$ _____ $\frac{18}{9}$

b. $-\frac{21}{12}$ _____ $-\frac{28}{16}$

c. $\frac{8}{20}$ _____ 0.35

d. $3\frac{5}{6}$ _____ $3\frac{7}{8}$

e. $\left|-\frac{3}{10}\right|$ _____ $\frac{3}{10}$

f. $\frac{12}{8}$ _____ $\frac{13}{9}$

3. From each pair of numbers, which number is to the left of the other on the number line?

   a. 3.5 , 7 b. $\frac{6}{8}$, $\frac{5}{7}$ c. $3\frac{2}{5}$, $2\frac{5}{7}$

   d. -9 , $\frac{13}{5}$ e. $-\frac{1}{3}$, -1.5

4. k, n, m, x, y, z are natural numbers represented on a number line as follows:

Figure 1. 14

Compare the numbers using > or <.

a. n____x b. x____y c. z____n d. y____k

1.2.2. Ordering Rational numbers

Ordering rational numbers means writing the given numbers in either ascending or descending order. Ordering a rational numbers of different denominators is a little bit like ordering distance measured in miles and kilometers, where we need all the distances to be in the same unit. For fractions, we need either to rewrite them in such a way that all have the same denominator or to convert them to decimals.

Example 1.14:

Arrange the following rational numbers in:

i) Increasing (ascending) order

a. $-25, 18, -45, 30, 28$

b. $\frac{3}{5}, \frac{4}{9}, \frac{7}{3}, \frac{5}{6}, \frac{11}{18}$

c. $2.5, 1.5, 3.21, 1.53, 2.05$

ii) Decreasing (descending) order

a. $-15, 45, 32, -23,$

b. $\frac{7}{3}, 2\frac{4}{5}, \frac{5}{2}, \frac{8}{15},$

c. $4.5, 5.17, 3.75, 4.75$

Solution:

i) a. $-25 < -45 < 18 < 28 < 30$

b. To order these rational numbers, first change the fractions with the same denominator.

Thus, $\frac{54}{90}, \frac{40}{90}, \frac{210}{90}, \frac{75}{90},$ and $\frac{55}{90}$.

Compare only the numerators: $40 < 54 < 55 < 75 < 210$

Therefore, the numbers in increasing order are

$$\frac{4}{9} < \frac{3}{5} < \frac{11}{18} < \frac{5}{6} < \frac{7}{3}$$

c. $1.5 < 1.53 < 2.05 < 2.5 < 3.21$

ii) a. $45 > 32 > -15 > -23$

b. To order these rational numbers, first change the fractions with the same denominator. thus $\frac{70}{30}, \frac{84}{30}, \frac{75}{30}, \frac{16}{30}$

Now compare only the numerators, 84 > 75 > 70 >16

Therefore, $2\frac{4}{5} > \frac{5}{2} > \frac{7}{3} > \frac{8}{15}$

c. 5.17 > 4.75 > 4.5 > 3.75

Exercise 1.5

1. Arrange the following rational numbers in ascending order.

   a. $\frac{4}{9}, \frac{3}{25}, \frac{11}{7}, -5\frac{2}{3}, \frac{23}{15}$

   b. 5.24, 8.13, 6.75, 12.42, −12.51

   c. 3.92 , $\frac{5}{13}$, $4\frac{6}{7}$, 4.73, $\frac{11}{9}$

2. Arrange the following rational numbers in descending order.

   a. 13.72, 23.86, 15.02, 13.05

   b. $\frac{21}{12}$, $\frac{13}{16}$, $2\frac{7}{9}$, $\frac{9}{7}$, $2\frac{4}{9}$

   c. $3\frac{5}{6}$, 3.75 , $\frac{18}{5}$, 4.23, 3.21

3. Samuel's science class is "growing plants under different conditions". The average plant growth during a week was 5.5cm.The table shows the difference from the average for some students' plants.

   a. Which student's plant growing more?

   b. Order the differences from lowest to highest.

	Difference from Average Plant Growth			
Student	Rahel	Kassa	Alemitu	Munir
Difference	$3\frac{1}{4}$	−2.2	1.7	$-1\frac{7}{10}$

1.3. Operation and properties of Rational Numbers

Competencies: At the end of this sub-unit students should:

- Add rational numbers.
- Subtract rational numbers.
- Multiply rational numbers
- Divide rational numbers.

1.3.1. Addition of rational numbers

Activity 1.3

1. Add the following numbers using a number line and show using arrows.

   a. $6+(-3)$ c. $2+(-4)$

   b. $-9+5$ d. $6+(-6)$

2. Find the sum of $\frac{3}{6}$ and $\frac{2}{6}$ using fraction bar.

Adding rational numbers with same denominators

To add two or more rational numbers with the same denominators, we add all the numerators and write the common denominator.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{b}$, $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

Example 1.15:

Find the sum of the following rational numbers

a. $\frac{3}{5}+\frac{6}{5}$

b. $\frac{8}{7}+\frac{5}{7}+\frac{6}{7}$

c. $-\frac{3}{4}+\frac{11}{4}$

d. $2\frac{1}{3}+\frac{4}{3}+1$

Solution:

a. $\frac{3}{5}+\frac{6}{5}=\frac{3+6}{5}=\frac{9}{5}$

b. $\frac{8}{7}+\frac{5}{7}+\frac{6}{7}=\frac{8+5+6}{7}=\frac{19}{7}$

c. $-\frac{3}{4}+\frac{11}{4}=\frac{-3+11}{4}=\frac{8}{4}=2$

d. $2\frac{1}{3}+\frac{4}{3}+1=\frac{7}{3}+\frac{4}{3}+\frac{3}{3}=\frac{7+4+3}{3}=\frac{14}{3}$

Adding rational numbers with different denominators

To find the sum of two or more rational numbers which do not have the same denominator, we follow the following steps:

I. Make all the denominators positive.

II. Find the LCM of the denominators of the given rational numbers.

III. Find the equivalent rational numbers with common denominator.

IV. Add the numerators and take the common denominator.

Example 1.16:

Find the sum of the following rational numbers.

a. $\frac{3}{7}+\frac{5}{4}$

b. $\frac{11}{9}+\frac{5}{6}+\frac{3}{4}$

Solution:

a. The LCM of 7 and 4 is 28.

Then, write the fraction as a common denominator 28.

$\frac{3}{7}\times\frac{4}{4}=\frac{12}{28}$ and $\frac{5}{4}\times\frac{7}{7}=\frac{35}{28}$

$\frac{3}{7}+\frac{5}{4}=\frac{12}{28}+\frac{35}{28}=\frac{12+35}{28}=\frac{47}{28}$

b. The LCM of 9 , 6 and 4 is 36.

Then, write the fraction as a common denominator 36.

$\frac{11}{9}\times\frac{4}{4}=\frac{44}{36}$, $\frac{5}{6}\times\frac{6}{6}=\frac{30}{36}$ and $\frac{3}{4}\times\frac{9}{9}=\frac{27}{36}$

$$\frac{11}{9}+\frac{5}{6}+\frac{3}{4}=\frac{44}{36}+\frac{30}{36}+\frac{27}{36}=\frac{44+30+27}{36}=\frac{101}{36}$$

Note: Always reduce your final answer to its lowest term.

Now you are going to discover some efficient rules for adding any two rational numbers.

Rule 1: To find the sum of two rational numbers where both are negatives:

i) Sign : Negative $(-)$

ii) Take the sum of the absolute values of the addends.

iii) Put the sign in front of the sum.

Example 1.17:

Perform the following operation:

$$-\frac{8}{6}+(-\frac{11}{8})$$

Solution:

$$-\frac{8}{6}+(-\frac{11}{8})$$

i. Sign $(-)$

ii. $\left|-\frac{8}{6}\right|+\left|-\frac{11}{8}\right|=\frac{8}{6}+\frac{11}{8}=\frac{65}{24}$

Therefore, $-\frac{8}{6}+(-\frac{11}{8})=-\frac{65}{24}$

Rule 2: To find the sum of two rational numbers, where the signs of the addends are different, are as follows:

i) Take the sign of the addend with the greater absolute value.

ii) Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.

iii) Put the sign in front of the difference.

Example 1.18:

Perform the following operation

a. $-\frac{9}{4}+\frac{5}{4}$

b. $-\frac{5}{6}+\frac{3}{4}$

Solution:

a. $-\frac{9}{4}+\frac{5}{4}$

i. Sign$(-)$ because $\left|-\frac{9}{4}\right| > \left|\frac{5}{4}\right|$

ii. Take the difference of the absolute values:

$$\left|-\frac{9}{4}\right|-\left|\frac{5}{4}\right| = \frac{9}{4}-\frac{5}{4} = \frac{4}{4} = 1$$

Therefore, $-\frac{9}{4}+\frac{5}{4} = -\frac{4}{4} = -1$

b. $-\frac{5}{6}+\frac{3}{4}$

i. Sign$(-)$ because $\left|-\frac{5}{6}\right| > \left|\frac{3}{4}\right|$

ii. Take the difference of the absolute values:

$$\left|-\frac{5}{6}\right|-\left|\frac{3}{4}\right| = \frac{5}{6}-\frac{3}{4} = \frac{1}{12}$$

Therefore, $-\frac{5}{6}+\frac{3}{4} = -\frac{1}{12}$

Example 1.19:

Find the sum of $\frac{5}{8}$ and $\frac{2}{8}$ using fraction bar.

Solution:

Divide the fraction bar into 8 equal parts. Now shade 5 of them with gray color and 2 of them with green color as shown in the figure below:

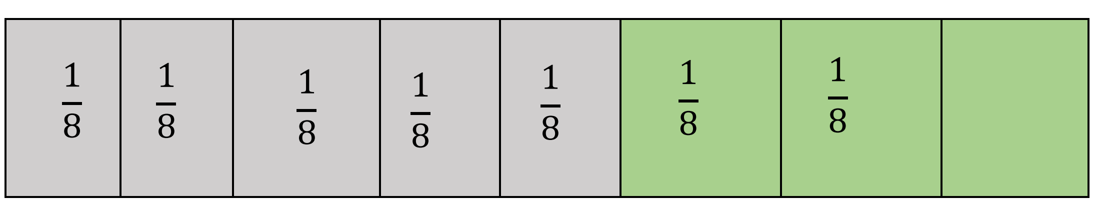


Figure 1.15

The shaded part within gray color represents $\frac{5}{8}$ and within green color represents $\frac{2}{8}$ of the whole fraction bar. The shaded part in both colors is 7 of eight equal parts. This shows that $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

Example 1.20:

Find the sum of $\frac{1}{6}$ and $\frac{1}{2}$ using fraction bar.

Solution:

Divide the fraction bar into 6 equal parts and shade one of them, which represents $\frac{1}{6}$. Similarly, Shade the other three parts in different colors which represents $\frac{3}{6} = \frac{1}{2}$.

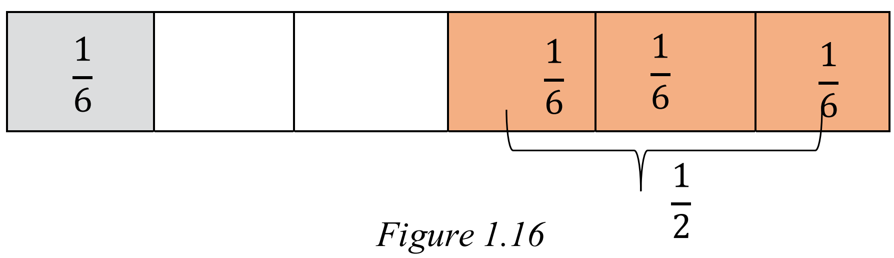


Figure 1.16

How many parts of the fraction bar shaded? 4 parts of the 6 equal parts. This implies that $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

Properties of Addition of Rational Numbers

For any rational numbers a, b and c the following properties of addition holds true:

a. **Commutative:** $a + b = b + a$

b. **Associative:** $a + (b + c) = (a + b) + c$

c. **Properties of 0:** $a + 0 = a = 0 + a$

d. **Properties of opposites** : $a + (-a) = 0$

Example 1.21:

Using properties of addition find the sum: $\frac{1}{3} + \frac{3}{5} + \frac{7}{6}$

Solution:

$\frac{1}{3}+\frac{3}{5}+\frac{7}{6} = (\frac{1}{3}+\frac{3}{5}) + \frac{7}{6}$ -------- Associative property

$= (\frac{5\ +\ 9}{15}) + \frac{7}{6}$

$= \frac{14}{15} + \frac{7}{6}$

$= \frac{28\ +35}{30} = \frac{63}{30}$

$= \frac{21}{10}$

Similarly, $\frac{1}{3}+\frac{3}{5}+\frac{7}{6} = \frac{1}{3} + (\frac{3}{5}+\frac{7}{6})$ ------ Associative property

$= \frac{1}{3} + (\frac{18+35}{30})$

$= \frac{1}{3} + \frac{53}{30}$

$= \frac{10+53}{30}$

$= \frac{63}{30} = \frac{21}{10}$

Therefore, $(\frac{1}{3} + \frac{3}{5}) + \frac{7}{6} = \frac{1}{3} + (\frac{3}{5} + \frac{7}{6})$

Exercise 1.6

1. Find the sum:

a. $\frac{13}{5} + \frac{21}{5}$

b. $\frac{5}{6} + \frac{3}{8}$

c. $\frac{3}{5} + 2\frac{3}{5}$

d. $2\frac{1}{3} + \frac{3}{8} + 3\frac{5}{6}$

e. $\left|-\frac{2}{5} + \frac{3}{8}\right| + \left|\frac{4}{7} + \frac{2}{7}\right|$

2. In the city where I live, the temperature on an outdoor thermometer on Monday was 23.72°c. The temperature on Thursday was 3.23°c more than that of Monday. What was the temperature on Thursday?

1.3.2. Subtraction of rational numbers

Activity:1.4

1. Can you express subtraction of rational numbers in the form of addition?
2. Are the commutative and associative properties holds true in subtraction of rational numbers?

The process of subtraction of rational numbers is the same as that of addition.

Subtraction of any rational numbers can be explained as the inverse of addition:

That is, for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, subtracting $\frac{c}{d}$ from $\frac{a}{b}$ means adding the negative of $\frac{c}{d}$ to $\frac{a}{b}$.

Thus $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + (-\frac{c}{d})$

Example 1.22:

Compute the following difference.

a. $\frac{7}{9} - \frac{4}{3}$ b. $\frac{3}{7} - (-\frac{5}{9})$ c. $-\frac{1}{12} - \frac{9}{8}$

Solution:

a. $\frac{7}{9} - \frac{4}{3} = \frac{7}{9} + (-\frac{4}{3})$

$= \frac{7+(-12)}{9}$

$= -\frac{5}{9}$

b. $\frac{3}{7} - (-\frac{5}{9}) = \frac{3}{7} + \frac{5}{9}$

$= \frac{27+35}{63}$

$= \frac{62}{63}$

c. $-\frac{1}{12} - \frac{9}{8} = -\frac{1}{12} + (-\frac{9}{8})$

$= \frac{-2+(-27)}{24}$

$= -\frac{29}{24}$

Note:

i. The difference of two rational numbers is always a rational number.

ii. Addition and subtraction are inverse operations of each other.

Example 1.23:

Find the difference of $\frac{7}{9} - \frac{2}{3}$ using fraction bar.

Solution:

Divide the fraction bar in to 9 equal parts and shade 7 parts of them. Out of the shaded, mark 6 of them with x. Now, 6 parts of the 9 equal parts $\frac{6}{9}$ represent $\frac{2}{3}$ of the fraction bar.

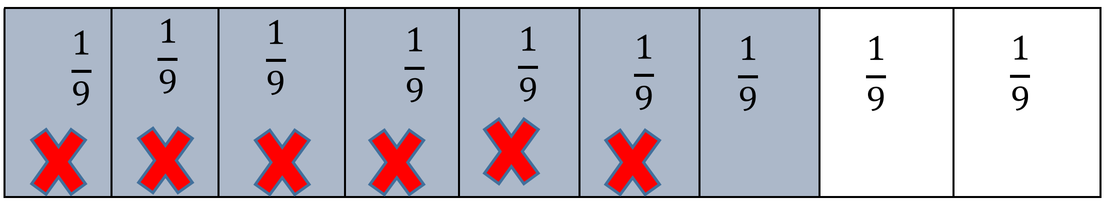


Figure 1.17

How many parts of the shaded part is unmarked? One part. This implies that $\frac{7}{9} - \frac{2}{3} = \frac{7}{9} - \frac{6}{9} = \frac{1}{9}$

Exercise 1.7

1. Find the difference of each of the following
   a. $4\frac{5}{6} - 2\frac{3}{4}$
   b. $-5.3 - 3.45$
   c. $\frac{6}{13} - \left| -\frac{7}{13} \right|$
   d. $\left| -\frac{5}{7} \right| - \left| \frac{3}{4} \right|$
   e. $-32.24 - \left| -32.24 \right|$
   f. $-3\frac{2}{5} - 2\frac{3}{7}$
2. Evaluate the following expressions:
   a. $y - (\frac{2}{7} + 5)$, when $y = \frac{9}{4}$
   b. $15 - (-y - \frac{4}{9})$, when $y = 7$

3. From a rope 23 m long, two pieces of lengths $\frac{12}{7}$ m and $\frac{7}{4}$ m are cut off. What is the length of the remaining rope?
4. A basket contains three types of fruits, apples, oranges and bananas, weighing $\frac{58}{3}$ kg in all. If $\frac{18}{7}$ kg be apples, $\frac{11}{9}$ kg be oranges and the rest are bananas. What is the weight of the bananas in the basket?

1.3.3. Multiplication of rational numbers

Activity 1.5

Multiply the following rational numbers

a. $\frac{5}{7} \times \frac{8}{11}$

b. $-\frac{9}{7} \times \frac{4}{5}$

c. $\frac{2}{3}\left(\frac{5}{9} \times \left(-\frac{3}{5}\right)\right)$

d. $3\frac{2}{5} \times 2\frac{4}{7}$

e) $-\frac{3}{8} \times \left(-\frac{2}{3}\right)$

To multiply two or more rational numbers, we simply multiply the numerator with the numerator and the denominator with the denominator. Finally reduce the final answer to its lowest term if it is.

Example 1.25:

Find the product

a) $\frac{3}{2} \times \frac{7}{8}$ b) $-\frac{5}{6} \times \frac{4}{3}$

Solution:

a) $\frac{3}{2} \times \frac{7}{8} = \frac{3 \times 7}{2 \times 8} = \frac{21}{16}$

b) $-\frac{5}{6} \times \frac{4}{3} = \frac{-5 \times 4}{6 \times 3} = \frac{-20}{18} = \frac{-10}{9}$

Example 1.25:

Find the product of $\frac{1}{2}$ and $\frac{3}{4}$ using grids model.

Solution:

a. model $\frac{1}{2}$ by shading half of a grid

i) Divide the grid in to 2 columns.

ii) Shade 1 column to show $\frac{1}{2}$

b. use a different color to shade $\frac{3}{4}$ of the same grid

i) Divide the grid in to 4 rows

ii) shade 3 rows to show $\frac{3}{4}$

$\frac{1}{2}$ 

Figure 1.18

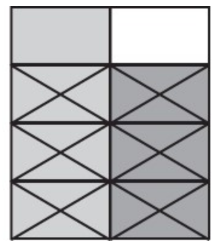
$\frac{3}{4}$ 

Figure 1.19

c. Determine what fraction of the grid is shaded with both colors.

There are 8 equal parts, and 3 of the part are shaded with both colors. The fraction shaded with both colors is $\frac{3}{8}$.

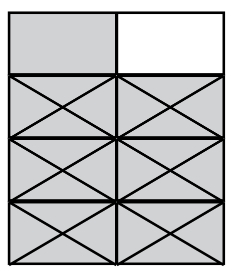
$\frac{3}{4}$ 

Figure 1.20

The section of the grid shaded with both colors shows 3 part of $\frac{1}{2}$ when $\frac{1}{2}$ is divided into 4 equal parts. In other words $\frac{3}{4}$ of $\frac{1}{2}$, or $\frac{3}{4} \times \frac{1}{2}$

Therefore, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

Note: The product of two rational numbers with different signs can be determine in three steps

1. Decide the sign of the product; it is "– ".
2. Take the product of the absolute value of the numbers.
3. Put the sign in front of the product.

Example 1.26:

Find the product

a. $\frac{4}{9}\times(-\frac{5}{2})$

b. $2\frac{5}{7}\times(-\frac{3}{2})$

Solution:

a. $\frac{4}{9}\times(-\frac{5}{2})$

i. Sign $(-)$

ii. Multiply the absolute value

$$\left|\frac{4}{9}\right|\times\left|-\frac{5}{2}\right|=\frac{4}{9}\times\frac{5}{2}=\frac{20}{18}=\frac{10}{9}$$

Therefore, $\frac{4}{9}\times(-\frac{5}{2})=-\frac{20}{18}=-\frac{10}{9}$

b. $2\frac{5}{7}\times(-\frac{3}{2})$

First change the mixed number to improper fraction.

$2\frac{5}{7}=2+\frac{5}{7}=\frac{19}{7}$, then

i. Sign $(-)$

ii. Multiply the absolute value

$$\left|\frac{19}{7}\right|\times\left|-\frac{3}{2}\right|=\frac{19}{7}\times\frac{3}{2}=\frac{57}{14}$$

Therefore, $2\frac{5}{7}\times(-\frac{3}{2})=-\frac{57}{14}$

Note: The product of two negative rational numbers is a positive rational number.

Example 1.27:

Find the product: $-\frac{2}{5}\times(-\frac{7}{3})$

Solution: $-\frac{2}{5} \times (-\frac{7}{3}) = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$

The following table 1.1 summarizes the facts about product of rational numbers.

The two factors	The product	Example
Both positive	**Positive**	$\frac{2}{3} \times 5 = \frac{10}{3}$
Both negative	**Positive**	$(-\frac{5}{4}) \times (-\frac{7}{3}) = \frac{35}{12}$
Of opposite sign	**Negative**	$-3 \times 5 = -15$
One or both 0	**Zero**	$-\frac{7}{9} \times 0 = 0$

Table 1.1

Properties of multiplication of rational numbers

For any rational numbers a, b and c ,the following properties of multiplication holds true:

a. Commutative: $a \times b = b \times a$

b. Associative: $a \times (b \times c) = (a \times b) \times c$

c. Distributive: $a \times (b + c) = a \times b + a \times c$

d. Property of 0: $a \times 0 = 0 = 0 \times a$

e. Property of 1: $a \times 1 = a = 1 \times a$

Example 1.28:

Using the properties of multiplication, find the following products.

a. $\frac{3}{5} \times \frac{6}{7}$ **b.** $\frac{2}{3} \times \frac{4}{5} \times \frac{5}{2}$ **c.** $\frac{2}{5} \times (\frac{3}{7} + \frac{4}{3})$ **d.** $(\frac{8}{9} \times \frac{5}{4}) \times 0$ **e.** $2\frac{3}{5} \times 1$

Solution:

a. $\frac{3}{5} \times \frac{6}{7} = \frac{6}{7} \times \frac{3}{5} = \frac{18}{35}$

b. $(\frac{2}{3} \times \frac{4}{5}) \times \frac{5}{2}$ …..associative property … $\frac{2}{3} \times (\frac{4}{5} \times \frac{5}{2})$

$= \frac{8}{15} \times \frac{5}{2}$ $\qquad = \frac{2}{3} \times \frac{20}{10}$

$= \frac{40}{30}$ $\qquad = \frac{40}{30}$

$= \frac{4}{3}$ $= \frac{4}{3}$

c. $\frac{2}{5} \times (\frac{3}{7} + \frac{4}{3})$

$= \frac{2}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{3}$ ….. *distributive property*

$= \frac{6}{35} + \frac{8}{15}$

$= \frac{74}{105}$

d. $(\frac{8}{9} \times \frac{5}{4}) \times 0 = 0$…….property of 0

e. $2\frac{3}{5} \times 1 = 2\frac{3}{5}$ ………property of 1

Example 1.29:

Find the cost of $\frac{35}{9}$ m of cloth, if the cost of a cloth per meter is Birr $\frac{162}{4}$.

Solution:

Total cost $= \frac{35}{9} \times \frac{162}{4} = \frac{5670}{36}$ = Birr $\frac{315}{2}$ = Birr 157.5

Note: To get the product with three or more factors, we use the following properties:

a. The product of an even number of negative factors is positive.

b. The product of an odd number of negative factors is negative.

c. The product of a rational number with at least one factor 0 is zero.

Exercise 1.8

1. Determine the product

a. $\frac{5}{8} \times \frac{3}{4}$ b. $\frac{3}{7} \times (-\frac{3}{4})$ c. $-\frac{8}{7} \times (-\frac{3}{5})$

d. $2\frac{3}{5} \times 4\frac{2}{3}$ e. $-\frac{2}{3} \times \frac{5}{4} \times \frac{3}{2}$

f. $-3(5x + 5)$, when $x = \frac{3}{4}$

2. An airplane covers 1250 km in an hour. How much distance will it cover in $\frac{23}{6}$ hours?

3. Find the product of $\frac{1}{3}$ and $\frac{2}{5}$ using grid model.

1.3.4. Division of Rational Numbers

Activity1.6

1. How many groups of $\frac{3}{4}$ are in 12?
2. How many groups of $\frac{2}{5}$ are in $3\frac{3}{5}$?
3. Use grids to model $3\frac{1}{3} \div \frac{2}{3}$.

Example 1.30:

Find the quotient by dividing $4\frac{1}{3}$ by $\frac{2}{3}$ using grid model.

Solution:

Since $4\frac{1}{3} = 4 + \frac{1}{3}$, divide each 5 grids in to 3 equal parts.

Shade 4 grids and $\frac{1}{3}$ of a fifth grid to represent $4\frac{1}{3}$ and

Divide the shaded grids in to equal groups of 2.

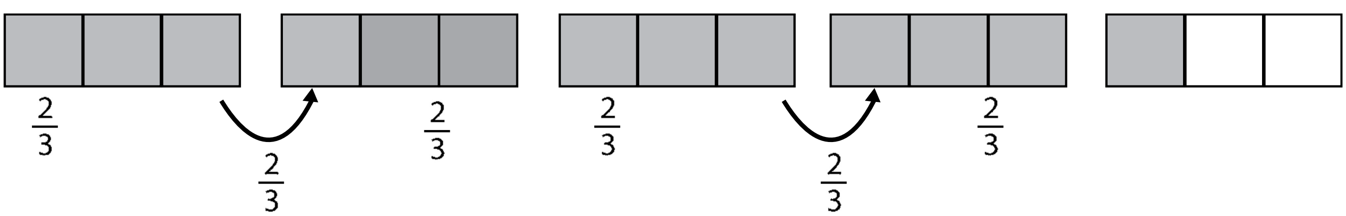


Figure 1.21

There are 6 groups of $\frac{2}{3}$, with in $\frac{1}{3}$ left over. This piece is $\frac{1}{2}$ of a group of $\frac{2}{3}$.

Thus, there are $6 + \frac{1}{2}$ groups of $\frac{2}{3}$ in $4\frac{1}{3}$.

Therefore, $4\frac{1}{3} \div \frac{2}{3} = 6 + \frac{1}{2} = \frac{13}{2}$

Note:

- $a \div b$ is read as a is divided by b.
- In $a \div b = c$, c is called the quotient, a is called the dividend and b is called the divisor.

- The quotient $a \div b$ is also denoted by $\frac{a}{b}$.
- If a, b and c are integers, $b \neq 0$ and $a \div b = c$, if and only if $a = c \times$

Rules for Division of Rational numbers

When dividing rational numbers:

1. Determine the sign of the quotient:
   a) If the sign of the dividend and the divisor are the same, then sign of the quotient is (+).
   b) If the sign of the dividend and the divisor are different, the sign of the quotient is (−).
2. Determine the value of the quotient by dividing the absolute value of the dividend by the divisor.

Example 1.31:

$$\frac{-324}{-18} = \frac{324}{18} = 18$$

Note: For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \text{ (where } c \neq 0)$$

Example 1.32:

Determine the quotient:

a. $\frac{6}{14} \div \frac{3}{7}$ b. $-3 \div \frac{9}{15}$ c. $-\frac{5}{7} \div -4$

Solution:

a. $\frac{6}{14} \div \frac{3}{7} = \frac{6}{14} \times \frac{7}{3} = \frac{6 \times 7}{14 \times 3} = \frac{42}{42} = 1$

b. $-3 \div \frac{9}{15} = -\frac{3}{1} \div \frac{9}{15} = -\frac{3}{1} \times \frac{15}{9} = -\frac{45}{9} = -5$

c. $-\frac{5}{7} \div -4 = -\frac{5}{7} \times -\frac{1}{4} = \frac{5}{28}$

Note: For any rational number $\frac{a}{b}$ where $a \neq 0$;

Exercise:1.9

1. Determine the quotient

a. $\frac{5}{8} \div \frac{3}{4}$

b. $-\frac{3}{5} \div \frac{6}{7}$

c. $-\frac{8}{9} \div (-\frac{5}{3})$

d. $2\frac{3}{5} \div (-\frac{4}{3})$

2. Rahel made $\frac{3}{4}$ of a pound of trail mix. If she puts $\frac{3}{8}$ of a pound into each bag, how many bags can Rahel fill?

$\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = \frac{ab}{ab} = 1$, Then $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

1.4. Real life applications of rational numbers

Rational numbers used to express many day –to- day real life activities. For instance, to share something among friends, to calculate interest rates on loans and mortgages, to calculate interest on saving accounts, to determine shopping discounts, to calculate prices, to prepare budgets, etc. So, in this sub-topic we will discuss some of them.

1.4.1. Application in sharing something among friends

Rational numbers are used in sharing and distributing something among a group of friends.

Example 1.33:

There are four friends and they want to divide a cake equally among themselves. Then, the amount of cake each friend will get is one fourth of the total cake.

Example 1.34:

Three brothers buy sugarcane. Their mother says that she will take over a fifth of the sugarcane. The brothers share the remaining sugarcane equally. What fraction of the original sugarcane does each brother get?

Solution:

Brother 1	Brother 2	Brother 3	1 2 3	Mother

Figure 1.22

Each brother gets a big piece $\frac{1}{5}$ and divides one big piece in to three equal parts, $\frac{1}{5} \div 3,$ which is $\frac{1}{15}$. so each brother gets:

$$\frac{1}{5} + \frac{1}{15} = \frac{3}{15} + \frac{1}{15} = \frac{4}{15}$$

1.4.2. Application in calculating interest and loans

Simple interest

Interest is a payment for the use of money or interest is the profit return on investment. Interest can be paid on money that is borrowed or loaned. The borrower pays interest and the lender receives interest. The money that is borrowed or loaned is called the principal (P). The portion paid for the use of money is called the interest (I). The length of time that money is used or for which interest is paid is called time (T). The rate per period (expressed as percentage) is called rate of interest (R). The interest paid on the original principal during the whole interest periods is called **simple interest**. Interest can be calculated by: I = PRT

Example 1.35:

Abebe borrowed Birr 21100 from CBE five months ago. When he first borrowed the money, they agreed that he would pay to CBE 15% simple interest. If Abebe pays to it back today, how much interest does he owe to it?

Solution:

Given Required

P =**B**irr 21100 I = ?

R = 15%

I = PRT

$\mathrm{I} = \text{Birr } 21100 \times 15\% \times \frac{5}{12}$, Where, T = 5 months = $\frac{5}{12}\,years$

$\mathrm{I} = \text{Birr } 21100 \times 0.15 \times \frac{5}{12}$

$\mathrm{I} = \text{Birr } 3165 \times \frac{5}{12}$

$\mathrm{I} = \text{Birr } 1318.75$

Therefore, Abebe pay an additional 1318.75 Birr of simple interest as per their agreement.

Example 1.36:

What principal would give Birr 250 interest in $2\frac{1}{2}$ years at a rate of 10%?

Solution:

Given Required

I = Birr 250 P =?

R = 10%

T = $2\frac{1}{2}$ years = 2.5 years

I = PRT

$P = \frac{\mathrm{I}}{RT} = \frac{\text{Birr } 250}{10\% \times 2.5}$

$P = \frac{\text{Birr } 250}{0.1 \times 2.5}$ $p = \text{Birr} 1000$

Exercise 1.10

1. If Birr 1200 is invested at 10% simple interest per annum, then What is simple interest after 5 years?
2. What principal will bring Birr 637 interest at a rate of 7% in 2 years?
3. Find the simple interest rate for a loan where Birr 6000 is borrowed and the amount owned after 5 months is Birr 7500.

SUMMARY FOR UNIT 1

1. A rational number is a number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.The set of rational numbers is denoted by $\mathbb{Q}$.
2. Rational numbers can represent on a number line.
3. The absolute value of a rational number 'x', denoted by $|x|$, is defined as:

$$|x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ x = 0 \\ -x, & if\ x < 0 \end{cases}$$

4. Rational numbers can be compared
   i. By changing to decimal numbers
   ii. By making the same denominators
   iii. By using cross product
5. Subtraction of any rational numbers can be explained as the inverse of addition.
6. The sum of two opposite rational numbers is 0.
7. Rules of signs of Addition
   a. The sum of two negative rational numbers is negative.
   b. The sum of negative and positive rational numbers is takes the sign of the greater absolute value.
8. Rules of signs of Multiplication
   a. The product of two negative rational numbers is positive.
   b. The product of negative and positive rational numbers is negative.
9. Rules of signs for division:
   a. positive divided by negative equals negative.
   b. negative divided by positive equals negative.
   c. negative divided by negative equals positive.

REVIEW EXERCISE FOR UNIT 1

1. Which of the following statements are true?

a. $-\frac{5}{3} > -\frac{8}{11}$

b. $3\frac{5}{7} > \frac{23}{7} > \frac{19}{6}$

c. $\left|2\frac{3}{5}\right| = \left|-2\frac{3}{5}\right|$

d. $3\frac{5}{6} < 3\frac{4}{5} < \frac{2}{7}$

2. Solve each of the following absolute value equations.

a. $|x| = 7$

b. $|x-5| = 4$

c. $|2x+3| = 5$

d. $2|3x-4| = 6$

e. $5+3|x-3| = 2$

3. If $x = -8$ and $y = 4$, then fined $\frac{3|x-5|-|4y|}{|x+y|}$

4. Find the sum

a. $\frac{3}{4}+\frac{9}{7}+2\frac{3}{5}$

b. $-2\frac{1}{3}+1\frac{4}{7}$

c. $3.35+2\frac{3}{7}$

5. Find the difference

a. $\frac{2}{7}-\frac{3}{8}$

b. $-2\frac{6}{7}-\frac{13}{9}$

6. Determine the product

a. $2\frac{3}{7}\times\frac{1}{4}$

b. $2.34\times\frac{7}{6}$

c. $-3\frac{3}{5}\times 1\frac{2}{3}\times(-\frac{3}{4})$

7. Determine the quotient

a. $\frac{3}{4}\div(-\frac{2}{7})$

b. $-3\frac{5}{8}\div 2\frac{3}{10}$

c. $2\frac{3}{7}\div 2.3$

8. Simplify the following expressions.

   a. $\frac{3}{5}+\frac{2}{7}(\frac{4}{5}+\frac{3}{2})$

   c. $\dfrac{\frac{1}{6}\div(\frac{1}{3}+\frac{4}{5})}{\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\div\frac{3}{5})}$

   b. $\frac{4}{3}\div(\frac{5}{2}-\frac{6}{7})$

9. Eleni baked a batch of 32 cupcakes and iced 24 of them. What fractions of cupcakes were iced?

10. How long will take Birr 500 to get Birr 50 simple interest at rate of 9.5%?

UNIT 2

SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

Learning outcomes: At the end of this unit, learners will be able to:

- Understand the notion square and square roots, and cubes and cube roots.
- Determine the square of numbers
- Determine the square roots of the perfect square numbers
- Extract the approximate square roots of numbers by using the numerical table and scientific calculator
- Determine the cube of numbers
- Extract the cube roots of perfect cubes
- Apply squares, square roots, cubes and cube roots in the real-life situation

Main Contents

2.1. Squares and Square Roots

2.2. Cubes and Cube roots

2.3. Applications on squares, square roots, cubes and cube roots

Summary

Review Exercise

INTRODUCTION

What you had learnt in the previous grades about multiplication will be used in this unit to describe special products known as squares and cubes of the given numbers. For instance, in a real-life application when you want to find a new apartment, you need to know the size of it. Newspaper and online advertisements usually give the dimensions of the apartment by only listing its area, such as 625 square meters. This can be hard to you use the concepts of square roots and convert this number in the form of 25×25 square meters. It gives better ideas of the apartment. The following sub-topics will give a brief explanation about squares and cubes.

2.1. Squares and Square roots

In this sub-topic you will learn about raising a given number to the power of "2" and extracting square roots of some perfect squares.

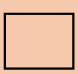
2.1.1. Square of a rational number

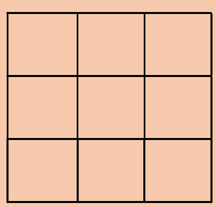
Competency: At the end of this sub-topic, students should:

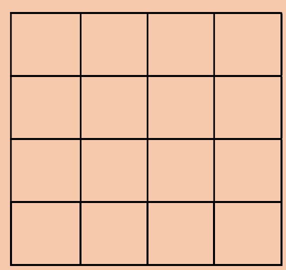
- Calculate the square of a number

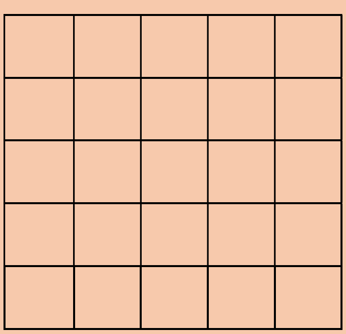
Activity 2.1

1, How many numbers of squares do you have?

a) 

b) c) 



d) 

2, Find the products of each of the following

a. $4 \times 5 =$ ______ c. $2.5 \times 6 =$ ______

b. $3\frac{1}{2} \times 2.5 =$ __________

3, Compute the products for each of the following.

a) $6 \times 6 =$ ______

b) $10 \times 10 =$ ______

c) $0.5 \times 0.5 =$ _____

Definition 2.1: The process of multiplying a number by itself is called squaring of a number.

Example 2.1.

a) $1 \times 1 = 1 = 1^2$

b) $2 \times 2 = 4 = 2^2$

c) $4 \times 4 = 16 = 4^2$

d) $5 \times 5 = 25 = 5^2$

Note :

If the number "a" to be multiplied by itself, then the product is usually written a^2 and read as:

- ✓ a *squared or*
- ✓ *The square of* or
- ✓ a *to the power of* 2

Example 2.2:

Read the following numbers

a. 2^2 b. 6^2 c. 10^2

Solution

a. 2^2 read as 2 squared or the square of 2 or 2 to the power of 2

b. 6^2 read as 6 squared or the square of 6 or 6 to the power of 2

c. 10^2 read as 10 squared or the square of 10 or 10 to the power of 2

Example 2.3:

Find the square of each of the following numbers

a) 8
c) 20
b) $\frac{4}{3}$
d) $-\frac{10}{16}$

Solution:

a) $8^2 = 8 \text{ x } 8 = 64$

b) $(\frac{4}{3})^2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

c) $20^2 = 20 \times 20 = 400$

d) $\left(-\frac{10}{16}\right)^2 = -\frac{10}{16} \times -\frac{10}{16} = \frac{100}{256}$

Group Work 2.1

Discuss the following patterns with your groups:

1[first odd number] $= 1 = 1^2$

$1 + 3$[sum of the first two odd numbers] $= 4 = 2^2$

$1 + 3 + 5$[sum of the first three odd numbers] $= 9 = 3^2$

$1 + 3 + 5 + 7$[sum of the first four odd numbers] $= 16 = 4^2$

$\vdots$

$1 + 3 + 5 + \cdots + 19$ [sum of the first ten odd numbers]$= 100 = 10^2$

a) What is the sum of the first 15 odd numbers?

b) What is the sum of the first n odd numbers?

c) What do you generalize?

From the above group work, we generalize that the sum of the first n odd natural numbers is n^2.

Note : There is a difference between a^2 and $2a$.

i. $a^2 = a \times a$

ii. $2a = a + a$

Consider the following examples to see the difference

a. $10^2 = 10 \times 10 = 100\ while\ 10 \times 2 = 20$

b. $(0.4)^2 = 0.4 \times 0.4 = 0.16\ while\ 0.4 \times 2 = 0.8$

In general, $a^2 \neq 2a$ for any rational number a.

Definition 2.2: A rational number x is called a perfect square if and only if $x = m^2$ for some $m \in \mathbb{Q}$.

Example 2.4:

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $(\frac{1}{5})^2 = \frac{1}{25}$, $(\frac{1}{6})^2 = \frac{1}{36}$. Thus,

1,4, 9, 16, $\frac{1}{25}$ and $\frac{1}{36}$ are perfect squares.

Note

i. The square of rational numbers is also rational numbers.

ii. $0 \times 0 = 0$ Therefore, $0^2 = 0$

iii. For any rational numbers a and b, $(ab)^2 = a^2b^2$

iv. For any rational numbers a and b, $(\frac{a}{b})^2 = \frac{a^2}{b^2}$ where $b \neq 0$

Exercise 2.1

1. Determine whether each of the following statements is true or false.

a) $10^2 = 10 \times 10$

b) $8^2 = 8 \times 2$

c) $x^2 = 2x$, where $x \in \mathbb{Q}$

d) $-60^2 = 3600$

e) $(-30)^2 = 900$

2. Find x^2 in each of the following rational numbers

a) $x = 6$
b) $x = -20$
c) $x = 3\frac{1}{4}$
d) $x = -\frac{5}{4}$
e) $x = 0.03$
f) $x = 4.5$
g) $x = \frac{12}{4}$

3. List out a perfect square numbers from the given list.

$4,\ 9,\ 12,\ 16,\ 18,\ 24,\ 36,\ 1.6,\ 0.04,\ \frac{1}{4},\ \frac{1}{121},\ 0.01,\ 225$

4. Explain whether the numbers are a square number or not.

a) 144 b) 201 c) 324

5. Which of the following are the squares of even numbers?

a) 196 b) 441 c) 400 d) 324
e) 625 f) what do you conclude?

6. Which of the following are the squares of odd numbers?

a) 121 b) 225 c) 196 d) 484 e) 529
f) what do you conclude?

7. Evaluate

a. $(38)^2 - (37)^2$
b. $(75)^2 - (74)^2$
c. $(92)^2 - (91)^2$
d. $(105)^2 - (104)^2$
e. $(141)^2 - (140)^2$
f. $(218)^2 - (217)^2$.

What do you conclude from the pattern?

8. Express 64 as the sum of the first eight odd numbers.

Theorem 2.1: Existence theorem

For any rational number x, there is a rational number y $(y \geq 0)$ such that $x^2 = y$

Example 2.5:

Find the square of the following rational numbers by using the existence theorem.

a) $x = 12$ b) $x = 0.04$ c) $x = \frac{1}{2}$ d) $x = -\frac{1}{4}$

Solution:

a) $x = 12$. Then $y = x^2 = 12^2 = 144$

b) $x = 0.04$. Then $y = x^2 = (0.04)^2 = 0.0016$

c) $x = \frac{1}{2}$. Then $y = x^2 = (\frac{1}{2})^2 = \frac{1}{4}$

d) $x = -\frac{1}{4}$. Then $y = x^2 = (-\frac{1}{4})^2 = \frac{1}{16}$

How to approximate square of a number?

Rough calculation could be carried out for approximating and checking the results in squaring rational numbers. Such approximation depends on rounding off decimal numbers. Rounding is the process to estimate (approximate) particular number in context. To round a number look at the next digit in the right place, if the digit is less than 5, round down and if the digit is 5 or more than 5, round up. Rounding decimals is useful to estimate an answer easily and quickly.

Example 2.6:

Find the approximate value of x^2 in each of the following decimals

a) $x = 4.3$

b) $x = 0.026$

c) $x = 2.45$

Solution:

a) $x = 4.3 \approx 4$ [since $3 < 5$]

$x^2 = 4^2 = 16$

Therefore, $(4.3)^2 \approx 16$

b) $x = 0.026 \approx 0.03$

$x^2 = 0.03^2 = 0.0009$

Therefore, $(0.026)^2 \approx 0.0009$

c) $x = 2.45 \approx 2.5$

$x^2 = (2.5)^2 = 6.25$

Therefore, $(2.45)^2 \approx 6.25$

Exercise 2.2:

1.Determine whether each of the following statements is true or false

a) $0^2 = 2$

b) $10^2 > (10.05)^2 > 11^2$

c) $(9.9)^2 = 100$

2. Find the approximate value of x^2 if

a) $x = 4.2$ b) $x = 10.8$ c) $x = -8.7$ d) $x = 1.06$

2.1.2 Use of table values and scientific calculator to find squares of rational numbers

Competency: At the end of this sub-topic, students should:

- Calculate the square of a number

Activity 2.2

Discuss with your friends

1. Define scientific notation of a number by your own words.
2. Express the following numbers in scientific notation.

   a) 450 b) 0.0045 c) 84.3 d) 0.256 e) 0.05
3. Use table values of square to find x^2 for each of the following

   a) $x = 1.51$ c) $x = 3.25$ e) $x = 5.06$

   b) $x = 4.60$ d) $x = 5.29$ f) $x = 7.64$

To find the square of rational number when it is written in the form of a decimal is very tedious and time consuming work. To avoid this tedious and time consuming work a table of squares is prepared and presented in the numerical tables at the end of this book. In the table the first column headed by x lists numbers from $1.0 -$ 9.9, the remaining columns are headed respectively by the digits 0 to 9.

x	0	1	2	3	4	5	6	7	8	9
1.0										
1.1										
1.2										
.										
.										
.										
9.9										

Example 2.7:

By using table of squares evaluate $(3.24)^2$

Solution:

The way how we can find the square of rational numbers from the table as follows:

$step\ i$. Find the row which start with 3.2

ste . Move to right along the row until you get column under 4

$step\ i$. Read the number at the intersection of the row and the column.

x	0	1	2	3	4	5	6	7	8	9
3.1										
3.2					10.50					
3.3										

Exercise 2.3

1. Determine whether each of the following statement is true or false
   a) $(3.22)^2 = 10.30$
   b) $(3.56)^2 = 30.91$
   c) $(9.9)^2 = (9.801)^2$
2. If $(3.67)^2 = 13.47$ then find
   a) $(36.7)^2$ b) $(367)^2$ c) $(0.367)^2$
3. If $(8.435)^2 = x$ then determine each of the following in terms of x.
   a) $(84.35)^2$ b) $(0.8435)^2$
4. Find the square of the number 8.95
   a) Using rough calculation method
   b) Using numeral table value
   c) Using Scientific calculator

Hence $(3.24)^2 = 10.50$

From the table check the value of the following square numbers are true.

a) $(3.10)^2 = 9.610$

b) $(35.2)^2 = (3.52 \times 10)^2 = (3.52)^2 \times 10^2 = 12.39 \times 100 = 1239$

c) $(0.365)^2 = (3.65)^2 \times (10^{-1})^2 = 13.32 \times 0.01 = 0.1332$

Example 2.8:

Find the square of the number 6.75 using each techniques and compare your result.

Solution:

i. Using Rough(approximate value) Calculation

$6.75 \approx 7$

$(6.75)^2 \approx (7)^2 = 49$

$(6.75)^2 \approx 49$

ii. Using Value obtain from table.

a) Find the row which start with 6.7

b) Find the column headed by 5

c) Read the number, that $(6.75)^2$ at the intersection of the row and the column.

Therefore $(6.75)^2 \approx 45.56$

iii. Using Scientific calculator (Exact value)

Multiply (6.75) *by* (6.75)

$(6.75) \times (6.75) = 45.5625$

Therefore $(6.75)^2 = 45.5625$

2.1.3. Square Roots of a Rational number

Competency: At the end of this sub-topic, students should:

- Calculate the square root of perfect squares

Activity 2.3:

Find the prime factorization of the following numbers by using the factor tree method

a) 16 b) 32 c) 64 d) 256

e) 400 f) 81 g) 625

Definition 2.3: For any two rational numbers a and b; if $a^2 = b$,then a is called the square root of b.

Example 2.9:

a) The square root of 4 is 2 because $2^2 = 4$

b) The square root of 9 is 3 because $3^2 = 9$

c) The square root of 16 is 4 because $4^2 = 16$

d) The square root of $\frac{25}{64}$ is $\frac{5}{8}$ because $(\frac{5}{8})^2 = \frac{25}{64}$

Note

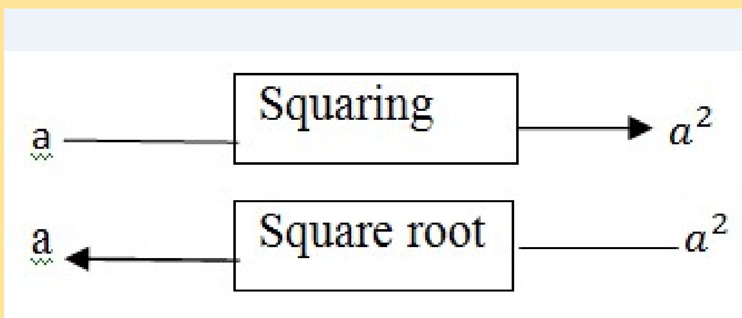
i. The operation "extracting square root" is the inverse of the operation squaring.

ii. In extracting square roots of rational numbers,
   - ✓ first decompose the number into product consisting of two equal factors and
   - ✓ take one of the equal factors as the square root of the given number.

iii. The positive square root of a number is called the principal square root. The symbol "$\sqrt{\ }$" called the radical sign, is used to indicate the principal square root.

iv. For $b \geq 0$, the expression $\sqrt{b}$ is called the principal square root of b or radical b, and b is called the radicand.

v. The relation between squaring and square root can be expressed as:



vi. In power form $\sqrt{a} = a^{\frac{1}{2}}$.

vii. Negative rational numbers don't have square roots in the set of rational number.

viii. The square root of zero is zero.

Example 2.10:

Find the square root of x, if x is

a) 16 b) 121 c) 0.04 d) $\frac{25}{100}$

Solution:

a) $x = 16 = 4 \times 4 \ = \ -4 \text{ x } -4$

$x = 4^2 = (-4)^2$

Thus, the square root of 16 is 4, $\sqrt{16} = 4$ [since "$\sqrt{\ }$" indicate the positive square root. To indicate the negative square root we use – "$\sqrt{\ }$" that is $-\sqrt{16} = -4$]

b) $x = 121 = 11 \times 11 = -11 \times -11$

$x = 11^2 = (-11)^2$

Thus, the square root of 121 is 11, $\sqrt{121} = 11$

c) $x = 0.04 = (0.2)^2 = (-0.2)^2$

Thus, the square root of 0.04 is 0.2, $\sqrt{0.04} = 0.2$

d) $x = \frac{25}{100} = \frac{5}{10} \times \frac{5}{10} = -\frac{5}{10} \times -\frac{5}{10}$

$x = (\frac{5}{10})^2 = (-\frac{5}{10})^2$

Thus the square root of $\frac{25}{100}$ is $\frac{5}{10}$, $\sqrt{\frac{25}{100}} = \frac{5}{10} = \frac{1}{2}$

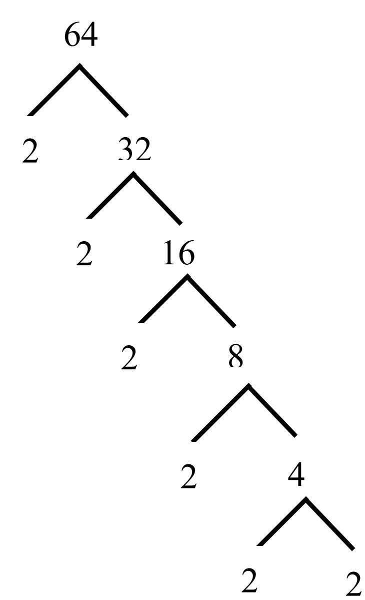
Example 2.11:

Find the square root of each of the following numbers by using prime factorization.

a) 64 b) 100 c) 400

Solution:

a) 64



Factorize 64 completely as shown to the right. Then arrange the factors so that 64 is the product of two identical factors.

i.e. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

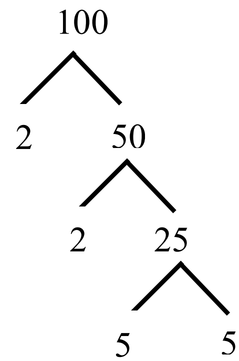
$64 = 8 \times 8$

$64 = 8^2$

so $\sqrt{64} = 8$

b) 100

Factorize 100 completely. Then arrange the factors so that 100 is the product of two identical factors



i.e. $100 = 2 \times 2 \times 5 \times 5$

$100 = (2 \times 5) \times (2 \times 5)$

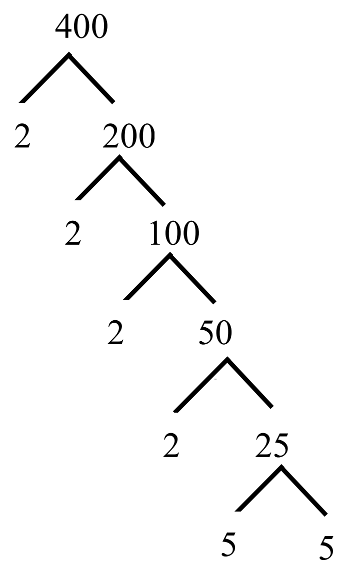
$100 = 10 \times 10$

$100 = 10^2$

So $\sqrt{100} = 10$

c) 400

factorize 400 completely as shown. Then arrange the factors so that 400 is the product of two identical factors



i.e. $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

$400 = 20 \times 20$

$400 = 20^2$

So $\sqrt{400} = 20$

Exercise 2.4

1. Evaluate the given square root.

a. $\sqrt{0}$ c) $\sqrt{576}$ e) $\sqrt{0.0016}$

b. $\sqrt{169}$ d) $\sqrt{0.25}$

2. The area of square is $144cm^2$.What is the length of each side?

3. Using prime factorization technique evaluates the square root of the following numbers.

a) 256 b) 324 c) 1225

2.1.4. Use of table values and scientific calculator to find square roots of rational numbers

Competency: At the end of this sub-topic, students should:

- Calculate the square root of a number.

Activity 2.4

Discuss with your friends

Use table value (Attached at the end of your text book) to evaluate the following square root.

a) $\sqrt{13.18}$ c) $\sqrt{11.16}$ e) $\sqrt{12.74}$

b) $\sqrt{98.01}$ d) $\sqrt{56.85}$

To find the square root of rational number when it is written in the form of a decimal is a tedious work. To avoid this tedious work a table of squares is prepared and presented in the numerical table at the end of this book.

Example 2.12:

Find $\sqrt{16.40}$ from the numeral table

Solution:

$step\ i$. Find the number 16.40 on the body of the table.

$step\ ii$. On the row containing this number move to the left and read 4.0 under x.

$step\ iii$. To get the third digit start from 16.40 move vertically up ward and read 5.

Therefore $\sqrt{16.40} \approx 4.05$

Note:

If the radicand is not found in the body of the table, you can approximate to the nearest square roots of a number.

Example 2.13:

Find $\sqrt{71.80}$ from numeral table

Solution:

$\sqrt{71.80}$ is not directly in the numerical table. So find two numbers from the table to the left and to the right of 71.80.

That is, $71.74 < 71.80 < 71.91$

Find the nearest number to 71.80 from those two numbers. Which is 71.74.

Thus $\sqrt{71.80} \approx \sqrt{71.74} = 8.47$

Therefore, $\sqrt{71.80} \approx 8.47$

Exercise 2.5

1.Find the square root of each of the following numbers from numerical table

a) 2.310 b) 4.326 c) 15.68

d) 98.60 f) 95.06

2. If $(4.63)^2 = 21.44$, then find

a) $\sqrt{21.44}$ b) $\sqrt{2144}$ c) $\sqrt{0.2144}$

2.2. Cubes and Cube roots

2.2.1. Cube of a rational number

Competency: At the end of this sub-topic, students should:

- Calculate the cube root of a number

Activity 2.5

1. Find x^3 in each of the following rational numbers.

a) $x = 2$ c) $x = -\frac{1}{4}$

b) $x = -4$ d) $x = 0.5$ e) 0

2. Which of those numbers are written as x^3?

a) 8 c) 121

b) 64 d) 729 e) 2700

Definition 2.4. A cube number is a number obtained by multiplying the number by itself three times.

Example 2.14:

The following are some cube numbers

a) $1 \times 1 \times 1 = 1$

b) $2 \times 2 \times 2 = 8$

c) $3 \times 3 \times 3 = 27$

d) $4 \times 4 \times 4 = 64$

Example 2.15:

Find x^3 in each of the following rational numbers

a. $x = 2$ b) $x = -4$ c) $x = 0.02$

d) $x = \frac{2}{3}$ e) $x = -\frac{1}{4}$

Solution:

a. $x^3 = x \times x \times x = 2 \times 2 \times 2 = 8$

b. $x^3 = x \times x \times x = -4 \times -4 \times -4 = -64$

c. $x^3 = x \times x \times x = 0.02 \times 0.02 \times 0.02 = 0.000008$

d. $x^3 = x \times x \times x = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

e. $x^3 = x \times x \times x = -\frac{1}{4} \times -\frac{1}{4} \times -\frac{1}{4} = -\frac{1}{64}$

Recall that: Rough calculation could be carried out for approximating and checking the results in cube of rational numbers. Such approximation depends on rounding off decimal numbers. Rounding is the process to estimating (approximating) particular number in context. To round a number, look at the next digit in the right place, if the digit is less than 5, round down and if the digit is 5 or more than 5, round up. Rounding decimals is useful to estimate an answer easily and quickly.

Example 2.16:

Find the approximate value of x^3 in each of the following decimals

a) $x = 3.2$

b) $x = 4.057$

c) $x = 10.57$

Solution:

a) $x = 3.2 \approx 3$

$x^3 = x \times x \times x \approx 3 \times 3 \times 3 = 27$

Therefore, $(3.2)^3 \approx 27$

b) $x = 4.057 \approx 4$

$x^3 = x \times x \times x \approx 4 \times 4 \times 4 = 64$

Therefore, $(4.057)^3 \approx 64$

c) $x = 10.57 \approx 11$

$x^3 = x \times x \times x \approx 11 \times 11 \times 11 = 1331$

Therefore, $(10.57)^3 \approx 1331$

Group Work 2.2

Discuss the following patterns with your groups

1[first odd number] $= 1 = 1^3$

$3 + 5$[the sum of the next two odd numbers] $= 8 = 2^3$

$7 + 9 + 11$[the sum of the next three odd numbers] $= 27 = 3^3$

$13 + 15 + 17 + 19$[the sum of the next four odd numbers]

$= 64 = 4^3$

$21 + 23 + 25 + 27 + 29$[the sum of the next five odd numbers]

$= 125 = 5^3$

$31 + 33 + 35 + 37 + 39 + 41$[the sum of the next six odd numbers]

$= 216 = 6^3$

$43 + 45 + 47 + 49 + 51 + 53 + 55$[the sum of the next seven odd numbers]

$= 343 = 7^3$

What will be the sum of the next eight odd numbers? (hint: see the above pattern)
How many consecutive odd numbers starting from 1 will be needed to obtain the sum as 100? 441?

Definition 2.5: A rational number x is called a perfect cube if and only if $x = m^3$ for some $m \in \mathbb{Q}$. That is, a perfect cube is a number that is a product of three identical factors.

Example 2.17:

The numbers $1, \frac{1}{8}, 27, -64, 125$ and 216 are perfect cubes. Because

$1 = 1^3$, $27 = 3^3$ $125 = 5^3$,

$\frac{1}{8} = (\frac{1}{2})^3$, $-64 = (-4)^3$, $216 = 6^3$

Exercise 2.6

1. Determine whether each of the following statement is true or false
   a) $2^3 = 2 \times 3$ b) $-20^3 = -400$ c) $8^3 = 64 \times 8$
   d) $(\frac{3}{4})^3 = \frac{27}{16}$ e) $(-3)^3 = -27$ f) $12^3 = 144 \times 12$
2. Find x^3 in each of the following
   a) $x = 4$ b) $x = 0.5$ c) $x = -\frac{1}{2}$ d) $x = \frac{3}{4}$
3. Find the approximate value of x^3 in each of the following
   a) $x = -3.45$ b) $x = 4.98$ c) $x = 0.025$
   d) $x = 2.75$
4. Identify whether each of the following are perfect cubes?
   a) 42 b) 60 c) $\frac{27}{64}$
   d) -125 e) 144 f) 216
5. What are the consecutive perfect cubes which added to obtain a sum of 100? 441?

2.2.2. Cube Root of a rational number

Competency: At the end of this sub-topic, students should:

- Calculate the cube of a number

Activity 2.6

1. Define a cube root of a number by your own words.
2. Find the cube root of each of the following numbers

   a) 512 b) 729 c) 343 d) 64 e) 1331

Definition 2.6: The cube root of a given number is one of the three identical factors whose product is the given number.

Example 2.18:

a) $1 \times 1 \times 1 = 1$, so 1 is the cube root of 1

b) $2 \times 2 \times 2 = 8$, so 2 is the cube root of 8

c) $4 \times 4 \times 4 = 64$, so 4 is the cube root of 64

Note:

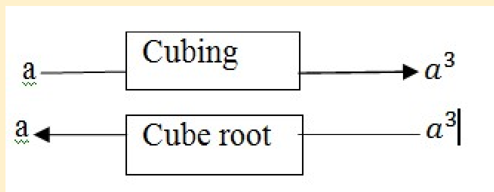
i. $3^3 = 27.$ Then 27 is the cube of 3 and 3 is the cube root of 27.

This is written as $3 = \sqrt[3]{27}$. The symbol $3 = \sqrt[3]{27}$ is read as the principal cube root of 27 or simply the cube root of 27.

ii. The symbol $\sqrt[3]{\ }$ is called a radical sign. The expression $\sqrt[3]{a}$ is called a radical, 3 is called the index and a is called the radicand.

When no index is written, the radical sign indicates square root.

iii. The relation between cubing and cube root can be expressed as:



iv. $\sqrt[3]{a} = a^{\frac{1}{3}}$ [exponential form]

v. Each rational number has exactly one cube root.

Exercise 2.7

1. State another name for $4^{\frac{1}{3}}$.
2. Write the following in exponential form

   a) $\sqrt[3]{10}$ b) $\sqrt[3]{0.23}$
3. Identify whether each of the following are perfect cube.

   3, 6, 8, 9, 12, 64, 216, 729, 625, 400
4. Find the cube root of the following numbers.

   a) 216 b) -343 c) 1000

   d) -1728 e) 0

2.3. Applications on squares, square roots, cubes and cube roots

Competency: At the end of this sub-topic, students should:

- Solve real-life problems

Activity 2.7

1. In figure 2.1 to the right

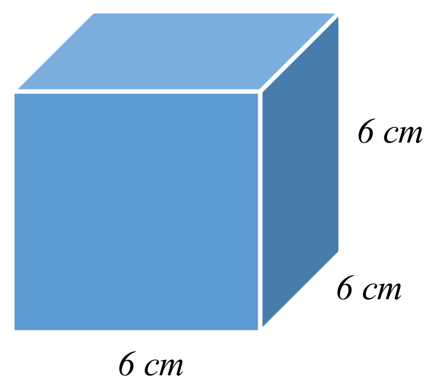
Find

a) The surface area of a cube

b) The volume of a cube

c) Compare the surface area and volume of a given cube.

2. Find the area of the square with side length 5cm ?



a) *Figure 2.1*

Squares, square roots, cubes and cube roots are used to express many day to day activities. For instance, squares and square roots are used in all walks of life, such as carpentry, engineering, designing buildings, and technology.

Cube root is used to solve for the dimensions of a three-dimensional object of a certain volume.

Example 2.19:

Alemu and Almaz want to make a square patio. They have concrete to make an area of 400 square meters. How long can a side of their patio be?

Solution:

Let $x$be the length of each sides of a square patio. Then,

$A = x^2$

$400 = x^2$. From this x can be calculated as principal square root:

$$x = \sqrt{400} = 20$$

Therefore, the length of each sides of a square patio is 20 m.

Example 2.19:

The length X of the sides of a cube is related to the volume V of a cube according to the formula: $X = \sqrt[3]{V}$. What is the volume of the cube if the side length is 8cm?

Solution:

$X = \sqrt[3]{V}$

$X^3 = V$

$8^3 = V$

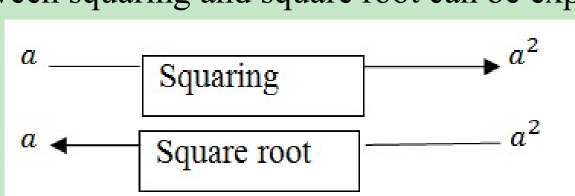
Therefore, $V = 512\text{cm}^3$

Exercise 2.8

1. 1225 students stand in rows in such a way that the number of rows is equal to the number of students in a row, how many students are there in each row?
2. Getaneh's flower garden is a square. If he enlarges it by increasing the width 1 m and the length 3 m, the area will be 19 square meters more than the present area. What is the length of a side now?
3. If the area of square region is 64 square meter, then what is the length sides of a square region?

SUMMARY FOR UNIT 2

1. The process of multiplying a number by itself is called squaring of a number.
2. A rational number x is called a perfect square if and only if $x = m^2$ for some $m \in \mathbb{Q}$
3. The square of rational numbers is also rational numbers.
4. $0 \times 0 = 0$ therefore $0^2 = 0$
5. For any rational numbers a and b, $(\mathrm{ab})^2 = \mathrm{a}^2\mathrm{b}^2$
6. For any rational number, a *and* b, $(\frac{\mathrm{a}}{\mathrm{b}})^2 = \frac{\mathrm{a}^2}{\mathrm{b}^2}$ where $b \neq 0$
7. For any rational number x,there is a rational number y $(y \geq 0)$such that $x^2 = y$
8. For any two rational numbers a and b if $a^2 = b$,then a is called the square root of $b, b \geq 0$
9. The operation "extracting square root" is the inverse of the operation squaring.
10. In extracting square roots of rational numbers ,
    - ✓ first decompose the number into product consisting of two equal factors and
    - ✓ Take one of the equal factors as the square root of the given number.
11. The positive square root of a number is called the principal square root. The symbol "$\sqrt{}$"called the radical sign, is used to indicate the principal square root.
12. For $b \geq 0$,the expression $\sqrt{b}$ is called the principal square root of b or radical b, b is called the radicand.
13. The relation between squaring and square root can be expressed as follow as



14. In exponent form $\sqrt{a} = a^{\frac{1}{2}}$.

15. Negative rational numbers don't have square roots in the set of rational number.

16. The square root of zero is zero.

17. A cube number is a number obtained by multiplying the number by itself three times.

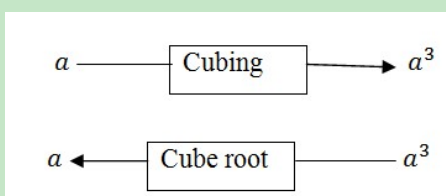
18. A rational number x is called a perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$ That is, a perfect cube is a number that is a product of three identical factors.

19. The cube root of a given number is one of the three identical factors whose product is the given number.

20. $4^3 = 64$. Then 64 is the cube of 4 and 4 is the cube root of 64. This is written as $4 = \sqrt[3]{64}$. The symbol $4 = \sqrt[3]{64}$ is read as the principal cube root of 64 or simply the cube root of 64.

21. The symbol $\sqrt[3]{\ }$ is called a radical sign, the expression $\sqrt[3]{a}$ is called a radical, 3 is called the index and a is called the radicand. When no index is written the radical sign indicates square root.

22. The relation between cubing and cube root can be expressed as:



23. $\sqrt[3]{a} = a^{\frac{1}{3}}$ [exponential form]

24. Each rational number has exactly one cube root.

REVIEW EXERCISE FOR UNIT 2

1. Determine whether each of the following statements is true or false

   a) $(0.8)^2 = 0.8 \times 2$ b) $-2^2 = -4$ c) $(-\frac{2}{3})^2 = \frac{4}{9}$

   d) $-3^3 = 27$ e) $0^3 = 3$ f) $(-5)^2 = 25$

2. Find x^2 in each of the following rational numbers

   a) $x = 0.3$ c) $x = \frac{1}{4}$

   b) $x = -2$ d) $x = -\frac{15}{4}$

3. Identify whether each of the following are perfect square.

   a) 216 b) 625 c) 1000

   d) 729 e) 900 f) 2025

4. What is the sum of the first 25 odd natural numbers?

5. Find the square of the number *8.04*

   a. Using rough calculation method

   b. Using numerical table value

   c. Using Scientific calculator

6. Find the square root of x if x is

   a) 64 b) 81 c) 0.04

7. Using prime factorization technique evaluates the square root of the following numbers.

   a) 225 b) 625 c) 2500

8. Find the square root of each of the following numbers from numeral table

   a)19.62 b) 10.56 c) 30.03 d) 64.64

9. If $(7.43)^2 = 55.20$, then find

   a) $(74.3)^2$ b) $(743)^2$ c) $(0.743)^2$

10. If $(3.42)^2 = 11.70$,then find

    a) $\sqrt{11.70}$ b) $\sqrt{1170}$ c) $\sqrt{0.117}$

11. Identify whether each of the following are perfect cubes?

a) 27 b) 60 c) 64 d) 25

12. Find x^3 in each of the following rational numbers

a) $x = 2$ b) $x = 0.03$

c) $x = -20$ d) $x = \frac{1}{4}$

13. Identify the base , exponent, power forms and Standard numeral form for each of the following numbers

a) $2^3 = 8$ b) $3^3 = 27$ c) $10^3 = 1000$

14. Write the following in power form

a) 900 b) 324 c) 216

15. Using the following pattern

1[first odd number] $= 1 = 1^3$

$3 + 5$[the sum of the next two odd numbers] $= 8 = 2^3$

$7 + 9 + 11$[the sum of the next three odd numbers]

$= 27 = 3^3$

$13 + 15 + 17 + 19$[the sum of the next four odd numbers]

$= 64 = 4^3$

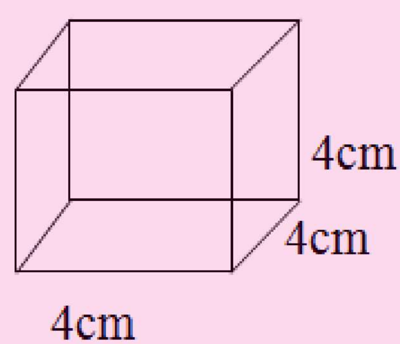
$21 + 23 + 25 + 27 + 29$[the sum of the next five odd numbers]

$= 125 = 5^3$

What will be the sum of six consecutive odd numbers next to 29?

16. In figure 2.2 as shown find:

a) The surface area of a cube
b) The volume of a cube
c) Compare the surface area and volume of a given cube.



17. What is the side length of the square with area $100cm^2$?

UNIT 3

LINEAR EQUATIONS AND INEQUALITIES

Learning outcomes: At the end of this unit, learners will be able to

- Graph linear equations of the type $y = mx + c$
- Solve linear inequalities.
- Solve application of linear inequalities.

Main Contents

3.1. Revision of Cartesian coordinate system

3.2. Graphs of Linear Equations

3.3. Solving Linear inequalities

3.4. Applications in Linear Equations and Inequalities

Summary

Review Exercise

INTRODUCTION

In grade 7, you have learnt about linear equations in one variable and the method to solve them, and graphs of $y = mx, m \in \mathbb{Q}, m \neq 0$. In this unit, we will discuss about linear inequalities and the methods to solve them and we sketch graphs of the form $y = mx + c, m \in \mathbb{Q}, m \neq 0$. Finally, we apply the knowledge about linear equations and inequalities to solve word problems.

3.1. Revision of Cartesian coordinate system

Competency: At the end of this sub-topic, students should:

- Describe the Cartesian coordinate system.

Activity 3.1

1. Describe the following terms in your own words.
   a. Cartesian coordinate plane
   b. Ordered pair
   c. Quadrant
2. Draw a pair of coordinate axes, and plot the points associated with each of the following ordered pair of numbers.

   A(0, 0) C(4, -3)

   B(-2, 1) D(0, 5)
3. In which quadrant are the following points located?

   A(7, -2) B(-3, 5) C(-9, -11)
4. Name the quadrant in which the point P(x, y) lies when:

   a) $x > 0, y < 0$ b) $x < 0, y < 0$

   c) $x < 0, y > 0$ d) $x > 0, y > 0$
5. Consider the equation $y = 3x - 1$.

   a) Determine the values of y when the values of x are -1, 0, 1

   b) Plot the ordered pairs on the Cartesian coordinate plane.

To determine the position of a point in a Cartesian coordinate plane, you have to draw two intersecting perpendicular number lines. The two intersecting perpendicular lines are called axes, the horizontal line is the x – axis and the vertical line is the y – axis. Usually the arrows indicate the positive direction. These axes intersect at a point called the origin. These two axes together form a plane called the Cartesian coordinate plane. The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These ordered pairs are called the coordinates of the point. The first coordinate is called the x- coordinate or abscissa and the second coordinate is called the y- coordinate or ordinate. The two axes divide the given plane into four quadrants. Starting from the positive direction of the x-axis and moving the anticlockwise (counter clock wise) direction, the quadrants which you come across are called the I, the II, the III, and the IV quadrants respectively.

Example 3.1:

The point with coordinates (2, 5) has been plotted on the Cartesian plane as follow. Imagine a vertical line through 2 on the x-axis and a horizontal line through 5 on the y-axis. The intersection of these two lines is the point (2, 5). This point is 2 units to the right of the y-axis and 5 units up from the x-axis.

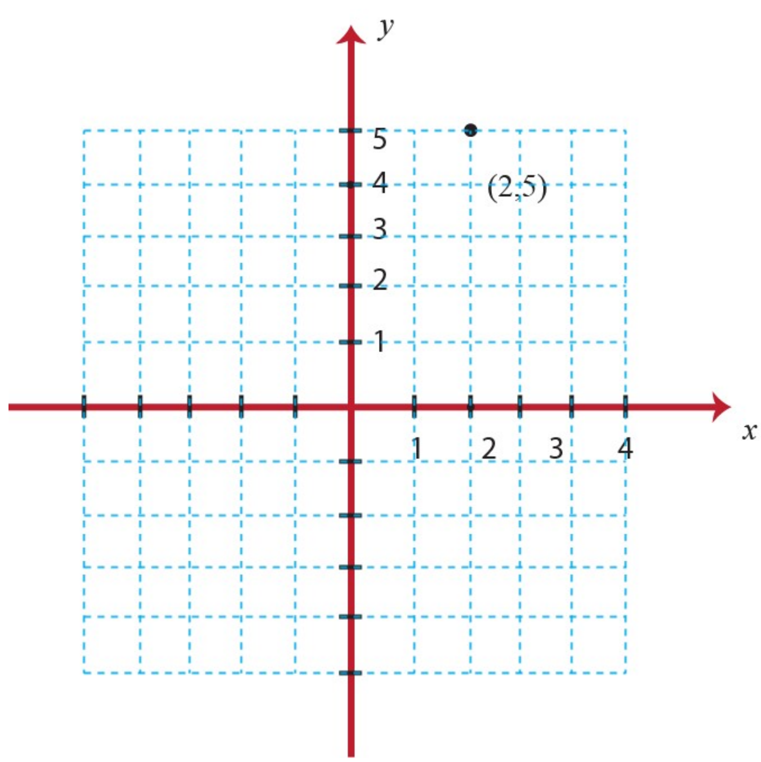


Figure 3.1

Note: In the I quadrant, all points are (+, +)

In the II quadrant, all points are (-, +)

In the III quadrant, all points are (-, -)

In the IV quadrant, all points are (+, -)

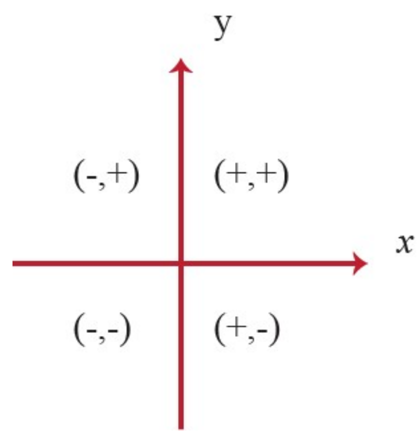


Figure 3.2

Example 3.2:

In which quadrant the following points lie?

a) $(2, -6)$ b) $(-3, 3)$ c) $(5, 0)$

Solution:

a) Since $x = 2 > 0$ and $y = -6 < 0$ then the point $(2, -6)$ lies in quadrant IV.

b) Since $x = -3 < 0$ and $y = 3 > 0$ then the point $(-3, 3)$ lies in quadrant II.

c) The point (5, 0) lies on the positive x- axis. It is neither of any quadrants.

Exercise 3.1:

1. Determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied?

a) $x > 0$ and $y < 0$

b) $x = -4$ and $y > 0$

c) $y < -5$

d) $x < 0$ and $-y > 0$

e) $x > 2$ and $y = 3$

f) $x > 0$

2. Find the coordinates of the point.

a) The point is located 5 units to the left of the y-axis and 2 units above the x-axis.

b) The point is located on the x-axis and 10 units to the left of the y-axis.

3.2: Graphs of Linear Equations

Competency: By the completion of this sub-topics, students should:

- Draw linear equations like $y = mx + c$ in a Cartesian coordinate plane

In this section, we will explore some basic principles for graphing.

Activity 3.2

1: Draw the graphs of the following lines.

a) x = 3 b) y = 3

2: write true if the statement is correct and false if it is incorrect

a) The line x = 5 is a vertical line.

b) The line y = -5 is a horizontal line.

c) The line 2x – 7 = 3 is a horizontal line.

d) The line 6 + 3y = -12 is a vertical line.

3: Write an equation representing

a) The x- axis

b) The y- axis

4: Evaluate: 5x – 2 when x = -3, x = 0, and x = 2

5: Solve for y if 3x + 5y = 4.

6: Let x = b. Where does the graph lies on the coordinate plane

a) If b < 0? b) If b = 0? c) If b > 0?

7: Let y = b. Which coordinate is constant, x-coordinate or y- coordinate? Which coordinate varies, x-coordinate or y- coordinate?

Revision on vertical and horizontal lines

In a vertical line all points have the same x- coordinate, but the y-coordinate can take any value. The equation of the vertical line through the point $P(a, b) is\ x = a$. This line is parallel to the y-axis and perpendicular to the x-axis. Similarly, in a horizontal line all points have the same y – coordinate but the x- coordinate can take any value. The equation of the horizontal line through the point $P(a, b)\ is\ y = b$. This line is parallel to the x-axis and perpendicular to the y-axis.

Example 3.3:

Sketch the graphs of

a) x = 5 b) y = -4

Solution: a) First construct tables of values for x and y in which x is constant.

x	5	5	5	5	5	5	5	5	5
y	-4	-3	-2	-1	0	1	2	3	4

Then, plot these points on the Cartesian coordinate plane and join them. What do you realize? All the points lie on the vertical line.

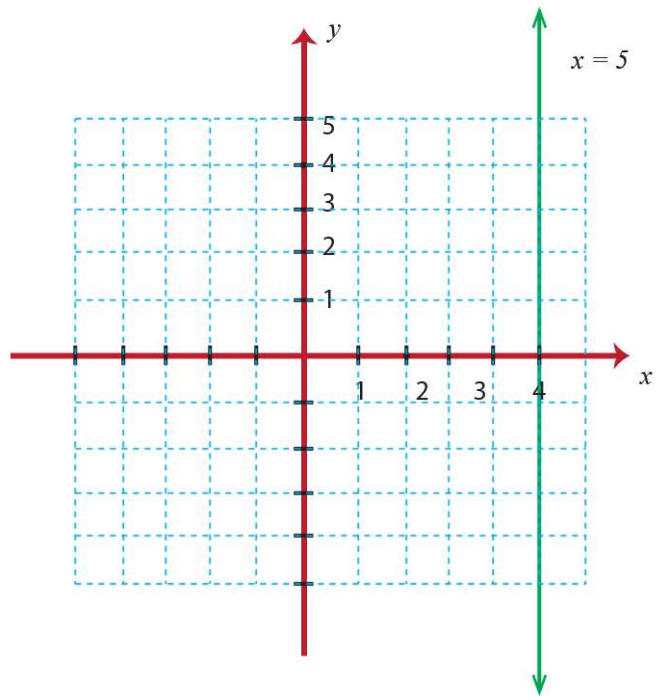


Figure 3. 3

b) First construct tables of values for x and y in which y is constant.

x	−5	−4	−3	−2	−1	0	1	2	3
y	−4	−4	−4	−4	−4	−4	−4	−4	−4

Then, plot these points on the Cartesian coordinate plane and join them. What do you realize? All points lie on the horizontal line.

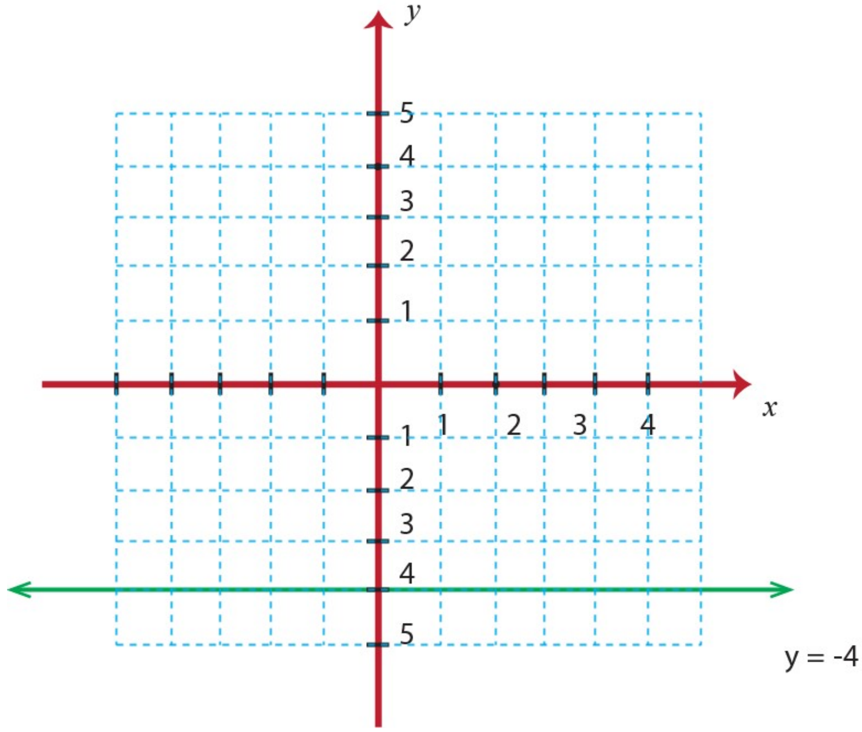


Figure 3.4

Graph of an equation of the form $\boldsymbol{y = mx}$ $(\boldsymbol{m \in \mathbb{Q}, m \neq 0})$

There are several methods that can be used to graph a linear equation. The method we used at the start of this section to graph is called plotting points, or the point – plotting method.

Activity 3.3

1) Define a linear equation

   a) in one variable. b) in two variables.

2) Which of the following is a linear equation in one variable?

   a) $3x + 1 = 4 - x$

   b) $x + 4 = \frac{1}{2}(2x + 3)$

c) $3(2x - 4) = 2(3x - 6)$

3) Refer question number 2,

a. How many solutions do you get for each equation?

b. What can you conclude about number of solutions for a linear equation in one variable?

4) Which of the following are linear equations in two variables?

a) $x = -5$

c) $y = \frac{1}{2}$

b) $x = 7 - 2y$

d) $3x + y = 4$

5) Given the equation $y = 4x$.

a) Are the points $(-1, 4)$, $(0, 0)$, $(2, 8)$ and $(3, 9)$ satisfy the equation?

b) When you plot points on the Cartesian coordinate plane, Which points lie on the same line?

c) Try to generalize about points on the line and solutions of the equation.

6) For which equation, does the line pass through the origin?

a) $y = 3$ b) $x = 2$ c) $y = -4x$ d) $y = 3x$

From the above activity, question number 5, we observe that for each value of x, there is one corresponding value of y. This relation is represented by an ordered pair (x, y). The set of all those ordered pairs that satisfy the equation $y = 4x$ is the solution of the equation $y = 4x$.

Graph of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

Definition 3.1:

The graph of a linear equation $= \boldsymbol{mx}$ $(\boldsymbol{m} \in \mathbb{Q}, \boldsymbol{m} \neq \mathbf{0})$, is a straight line passes through the origin.

Note:

- Every point on the line is a solution of the equation of a line.
- Every solution of the equation is a point on the line.

Example 3.4:

Sketch the graphs of the following equations on the same Cartesian coordinate plane

a) $y = 3x$ b) $y = -3x$

Solution:

To sketch the graphs of the equations, follow the following steps.

Step 1: Choose some values for x.

Step 2: Put these values of x into the equation to get the values of y.

Step 3: write these pairs of values in a table.

Step 4: plot the points on the Cartesian plane and join them.

Step 5: Label the line $y = mx$

a) $y = 3x$

when $x = -2,\ y = 3(-2) = -6$

when $x = -1,\ y = 3(-1) = -3$

when $x = 0,\ y = 3(0) = 0$

when $x = 1,\ y = 3(1) = 3$

when $x = 2,\ y = 3(2) = 6$

Step 3: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	-6	-3	0	3	6
(x, y)	$(-2,-6)$	$(-1,-3)$	(0,0)	(1, 3)	(2,6)

b) $y = -3x$

when $x = -2$, $y = -3(-2) = 6$

when $x = -1$, $y = -3(-1) = 3$

when $x = 0$, $y = -3(0) = 0$

when $x = 1$, $y = -3(1) = -3$

when $x = 2$, $y = -3(2) = -6$

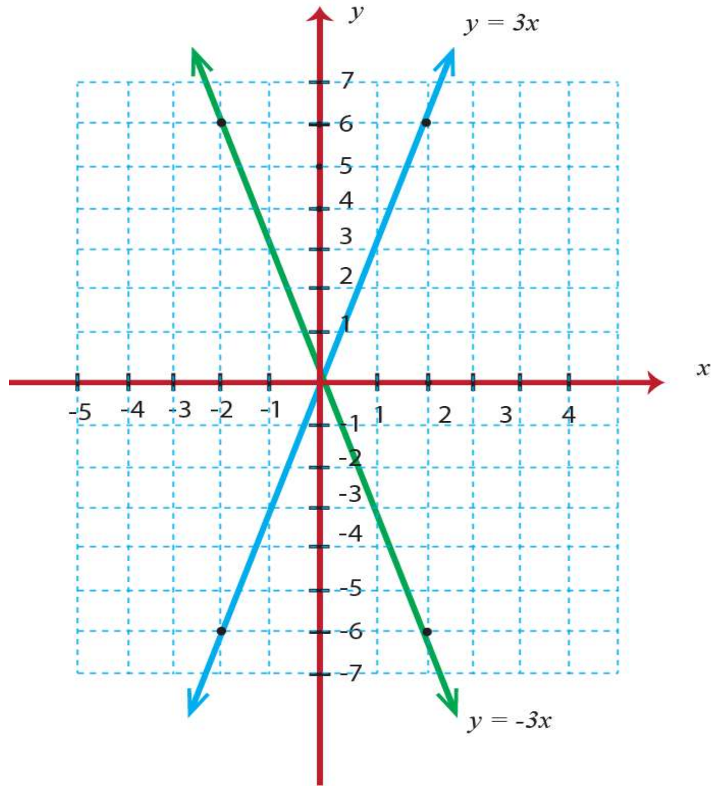


Figure 3.5

x	-2	-1	0	1	2
y	6	3	0	-3	-6
(x, y)	$(-2, 6)$	$(-1,3)$	$(0,0)$	$(1,-3)$	$(2,-6)$

If the point (k, 5) lies on the line $y = -3x$, then what is the value of k?

Since, the point is on the line, the point satisfies the equation of the line.

That is, $5 = -3(k)$

$$k = -\frac{5}{3}$$

Note:

- All ordered pairs that satisfy each linear equation of the form $\boldsymbol{y = mx}$ $(\boldsymbol{m} \in \mathbb{Q}, \boldsymbol{m} \neq \boldsymbol{0})$ lies on a straight line that passes through the origin.

- The graph of the line $\boldsymbol{y = mx}$ ($\boldsymbol{m \in \mathbb{Q}, m \neq 0}$) passes through the I and III quadrants if m > 0, and the graph passes through the II and IV quadrants if m < 0.
- In order to draw a straight line, you need to find any two points or coordinates through which the line passes.

Exercise 3.2:

1) Complete the following tables for sketching the graph of

   a) $y = -5x$ b) $y = \frac{1}{2}x$

   c) $y = x$ d) $y = -x$

x	-1	0	1	2
y				
(x, y)				

2) Sketch the graphs of the following equations on the same Cartesian coordinate plane.

   a) $y + 7x = 0$ b) $3y = 6x$

   c) $\frac{1}{4}x - \frac{y}{2} = 0$

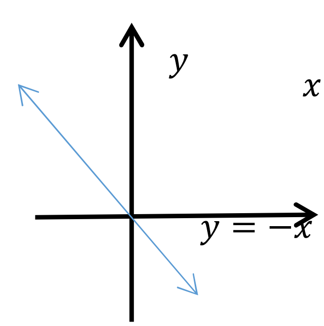
3) If the point P(a, 3) lies on the line given by the equation $12x - 2y = 0$, then find the value of a.

4) Answer the following questions below by referring the figure to the right.

   a) If $x = 3$, what is the value of y?

   b) If $y = -4$, what is the value of x?

   c) Can you find the point P(0, 0) exactly on the line?



$Figure$ 3.6

Figure

Graph of an equation of the form $y = mx + c$ ($m \in \mathbb{Q}, m \neq 0$)

Example 3.5:

1) Given that $y = 3x - 5$.Decide whether the ordered pairs given below are a solution to the equation.

a) $(0, -5)$ b) $(3, 4)$ c) $(-2, -11)$ d) $(-1, -2)$

Solution:

Substitute the x- and y- values into the equation to check whether the ordered pair is a solution to the equation.

a) $(0, -5)$

$y = 3x - 5.$
$-5 \,?\, 3(0) - 5$
$-5 = -5$
$(0, -5)$ is a solution.

b) $(3, 4)$

$y = 3x - 5.$
$4 \,?\, 3(3) - 5$
$4 = 4$
$(3, 4)$ is a solution.

c) $(-2, -11)$

$y = 3x - 5$

$-11 \,?\, 3(-2) - 5$

$-11 = -11$

$(-2, -11)$ is a solution.

d) $(-1, -2)$

$y = 3x - 5$

$-2 \,?\, 3(-1) - 5$

$-2 \neq -8$

$(-1, -2)$ is not a solution.

Example 3.6:

a) Sketch the graph of the equation y = 2x + 1 by plotting points.

Solution: Choose any value for x and solve for y.

Let x = -1	Let x = 0	Let x = 1
y= 2x +1	y= 2x + 1	y= 2x + 1
y = 2(0) + 1	y = 2(1) + 1	y= 2(-1) + 1
y= 1	y = 3	y = -1

Then, organize the solutions in a table

x	−1	0	1
y	−1	1	3
(x, y)	(−1, −1)	(0, 1)	(1,3)

Now, we plot the points on the Cartesian coordinate plane and draw the line through these points.

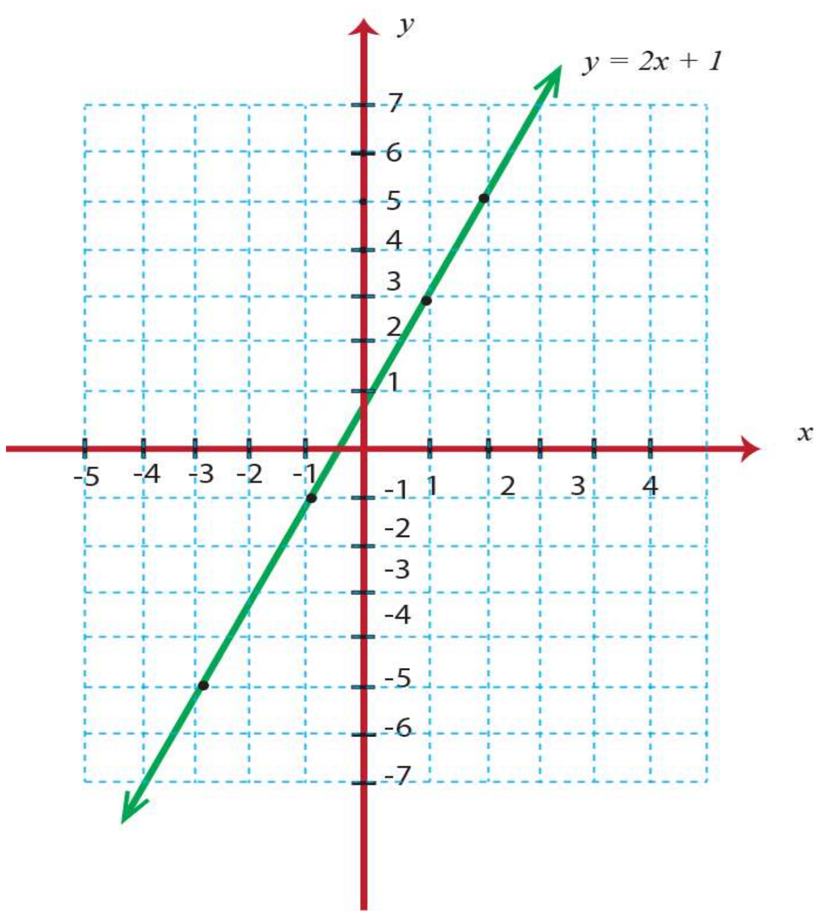


Figure 3.7

b) Sketch the graph of the equation $y = -2x + 1$ by plotting points.

Solution:

Choose any value for x and solve for y.

Let $x = -1$	Let $x = 0$	Let $x = 1$
$y = -2x + 1$	$y = -2x + 1$	$y = -2x + 1$
$y = -2(-1) + 1$	$y = -2(0) + 1$	$y = -2(1) + 1$
$y = 3$	$y = 1$	$y = -1$

Then, organize the solutions in a table

x	-1	0	1
y	3	1	-1
(x, y)	$(-1, 3)$	$(0, 1)$	$(1, -1)$

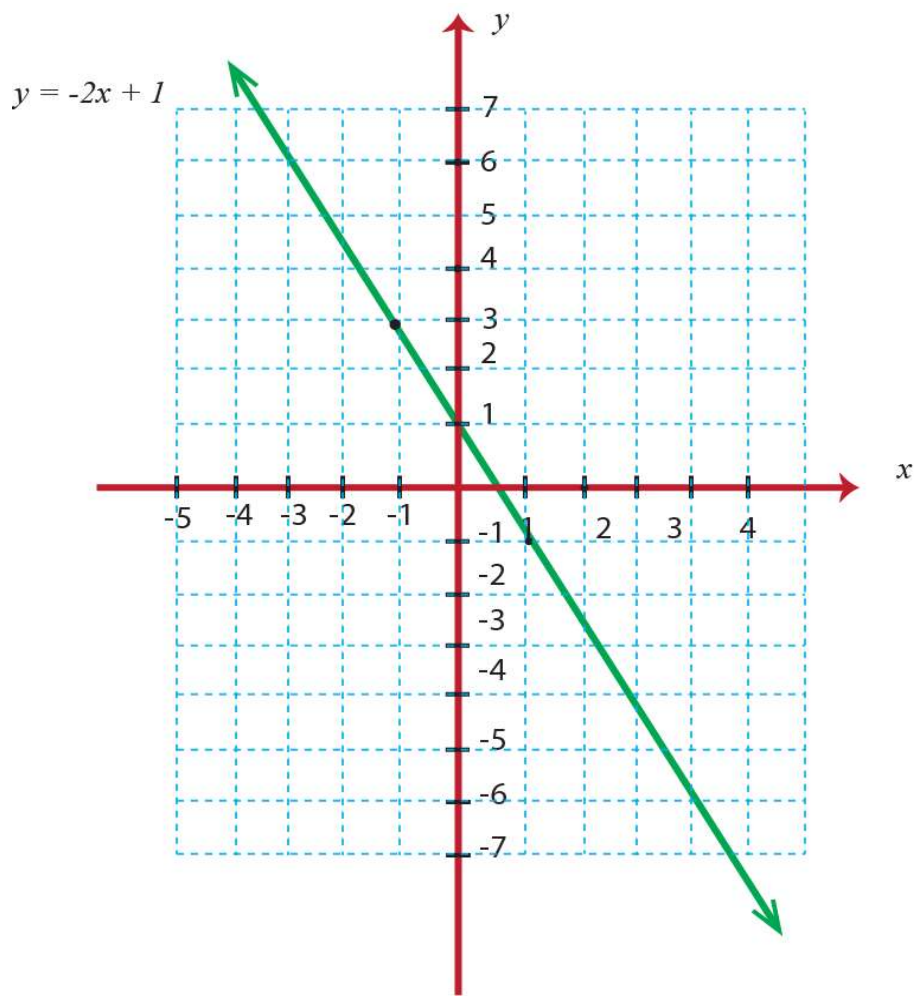


Figure 3.8

c) Sketch the graph of the equation $3y - 2x = 1$ by plotting points.

Solution: Choose any value for x and solve for y.

Let $x = -1$	Let $x = 0$	Let $x = 1$
$3y - 2x = 1$	$3y - 2x = 1$	$3y - 2x = 1$
$3y - 2(-1) = 1$	$3y - 2(0) = 1$	$3y - 2(1) = 1$
$y = -\frac{1}{3}$	$y = \frac{1}{3}$	$y = 1$

Then, organize the solutions in a table

x	-1	0	1
y	$-\frac{1}{3}$	$\frac{1}{3}$	1
(x, y)	$(-1, -\frac{1}{3})$	$(0, \frac{1}{3})$	$(1,1)$

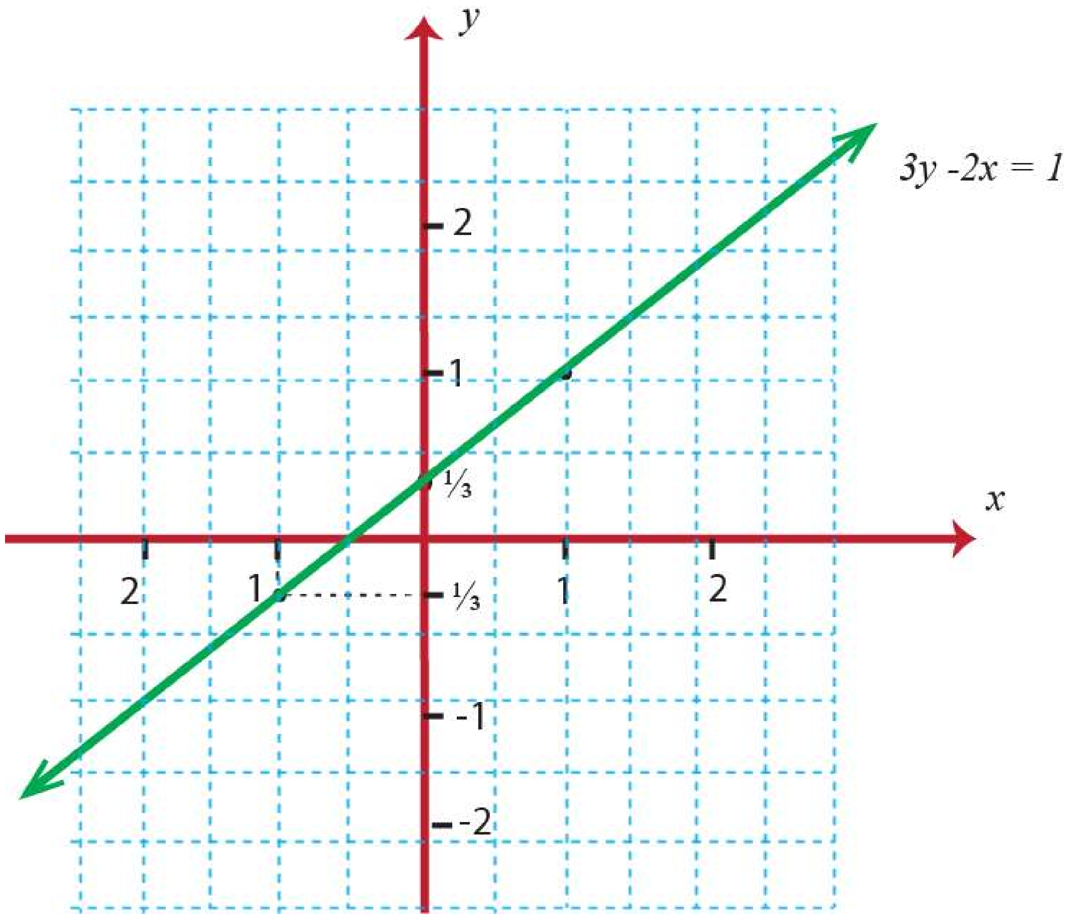


Figure 3.9

Exercise 3.3

Sketch the graph of the following equations.

a) $y = x + 3$

b) $2y - x = 6$

c) $y = x - 3$

d) $3y = x + 5$

3.3. Solving Linear Inequalities

Competency: At the end of this sub-topic, students should:

- Solve Linear Inequalities

In grade 7 mathematics lessons, you have learnt about linear equations. Now in this subtopic, you will learn more about linear inequalities.

Activity 3.4

1. Solve each of the following linear equations by the rules of transformation.
   a) $3(2x - 6) = 5x$
   b) $6x - 13 = 17 - (x + 2)$
   c) $0.5x + 0.5 = 0.2x + 2$
   d) $(3x - 1) - 4(2x + 1) = 4(4x - 3)$
2. Which of the following are linear inequalities?
   a) $-\frac{3}{2}x \leq 6x + 3$
   b) $2x \geq x + 2^2$
   c) $x^2 - 1 < \frac{1}{2}x - 5$
   d) $7x + \frac{1}{2}x = 4$
3. Solve each of the following linear inequalities in the set of W, $\mathbb{Z}$, $\mathbb{Q}$.
   a. $10x < 23$
   b. $-2x > 5$
   c) $\frac{1}{2}x \geq 4$
   d) $\frac{3}{2}x \leq 6$

Definition 3.2: A mathematical sentence which contains one of the relation signs: $<, >, \leq, \geq$ or $\neq$ are called inequalities.

Example 3.7:

Some examples of inequalities are:

a. $2x > 0$

b. $\frac{3}{5}x + 20 \leq 0$

c) $x - 1 \neq \frac{1}{2}x - 5$

d) $2x^3 - 1 < \frac{1}{2}x + 15$

Definition 3.3: A linear inequality in one variable "x" is an inequality that can be written in the form of $ax + b < 0$,

$ax + b \leq 0$ or $ax + b > 0$, $ax + b \geq 0$ where a and b are rational numbers and $a \neq 0$.

Example 3.8:

Some examples of linear inequalities are:

a) $x + 8 > 3$

b) $x - 0.35 \leq 0.25$

c) $4 - 3x \geq 1 + \frac{3}{2}x$

Solutions of Linear Inequalities

Consider the inequality $3x - 10 \leq 0$. If we substitute the variable x by a number, we will get a true or false mathematical statement.

For instance,

x	-3	-2	-1	0	1	2	3	4
$3x - 10$	-19	-16	-13	-10	-7	-4	-1	2

From the above table, we observe that $3x - 10 \leq 0$ is true for $x < 4$. This systematic trial (testing) method is tedious and also it cannot be always used to solve all types of problems. There is a more systematic method of solving inequalities. This method follows a procedure similar to the one we use for solving equations. This method depends on equivalent transformations.

Solving an inequality means applying the appropriate transformation rules to the given inequality get a value for the variable. The numbers that replace the variable and satisfy the inequality is called the solution of the inequality.

Rules of Transformation for Inequalities

There are rules that can help to transform a given true inequality to an equivalent inequality and finally at the solution set of the inequalities.

Definition 3.4: Any two inequalities with the same solution set are called **equivalent inequalities.**

Example 3.9:

$5x + 6 > 2x$ and $x > -2$ are equivalent inequalities.

The following rules are used to transform a given inequality to an equivalent inequality.

Rule 1: If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged.

That is for any rational numbers a, b and c.

i) If $a < b$, then $a + c < b + c$.

ii) If $a < b$, then $a - c < b - c$.

Rule 2: If both sides of an inequality are multiplied or divided by the same positive number, the direction of the inequality is unchanged.

That is for any rational numbers a, b and c.

i. If $a < b$ and $c > 0$, then $ac < bc$.

ii. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Rule 3: If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed.

That is for any rational numbers a, b and c.

i. If $a < b$ and $c < 0$, then $ac > bc$.

ii. If $a < b$ and $c < 0$, then, $\frac{a}{c} > \frac{b}{c}$.

Example 3.10:

Which of the following pairs of inequalities are equivalent inequalities in $\mathbb{Q}$?

a) $x + \frac{1}{2} < \frac{5}{2}$ and $x + 1 < 3$

b) $\frac{1}{2}x - 2x > 32$ and $x < -\frac{1}{2}$

Solution:

a) $x + \frac{1}{2} < \frac{5}{2}$

$x + \frac{1}{2} - \frac{1}{2} < \frac{5}{2} - \frac{1}{2}$ [Subtracting the same number from both sides]

$x < 2$ and

$x + 1 < 3$

$x + 1 - 1 < 3 - 1$ [Subtracting the same number from both sides]

$x < 2$

Since they have the same solution, $x + \frac{1}{2} < \frac{5}{2}$ and $x + 1 < 3$ are equivalent inequalities.

b) $\frac{1}{2}x - 2x > 32$

$2(\frac{1}{2}x - 2x) > 2 \times 32$

$x - 4x > 64$

$-3x > 64$

$-\frac{1}{3}(-3x) < -\frac{1}{3} \times 64$

$x < -\frac{64}{3}$

Therefore, $\frac{1}{2}x - 2x > 32$ and $x < -\frac{1}{2}$ are not equivalent inequalities.

Exercise 3.4:

1. Insert the correct sign ($<, >, \leq, \geq$ *or* $\neq$) in the given blank.

   a) If $x - 5 \geq 2x$, then x ------- -5

   b) If $\frac{1}{2}x + 2 \leq 3(x-1)$, then $\frac{5}{2}x$ ----- 5

   c) If $-3(x + 1) > 1$, then x + 3 ------ $\frac{5}{3}$

2. Which of the following pairs of inequalities are equivalent?

   a) $3x - 2 \leq x + 1$ and $2x + 1 \leq 4$

   b) $6(2 - x) < 12$ and $3(2 - x) > 6$

   c) $\frac{x}{4} - 1 \geq \frac{3}{2} - x$ and $x + 1 \geq 3$

   d) $5x - \frac{3}{7} \neq \frac{4}{7} - 3x$ and $x \neq \frac{1}{8}$

Solutions of Linear inequalities by means of equivalent transformations

The objective of this subtitle is to use the rules of transformation in solving linear inequalities by getting successive equivalent inequalities until we arrive at an inequality of the form $x < a$ or $x > a$ [or $x \leq a$ or $x \geq a$]

First, we will deal with simple inequalities which we need only one equivalent transformation to obtain the form $x < a$ or $x > a$

[or $x \leq a$ or $x \geq a$]

Example 3.11:

Solve each of the following inequality in the domain of $\mathbb{Q}$.

a) $x + 2.4 \leq 6.4$ b) $\frac{1}{2}x > \frac{5}{2}$

Solution:

a) $x + 2.4 \leq 6.4$

$x + 2.4 - 2.4 \leq 6.4 - 2.4$ [subtracting 2.4 from both sides]

$x \leq 4, x \in \mathbb{Q}$

b) $\frac{1}{2}x > \frac{5}{2}$

$2 \times \frac{1}{2}x > 2 \times \frac{5}{2}$ [Multiplying both sides by 2]

$x > 5, x \in \mathbb{Q}$

Next we will see inequalities which require more than one equivalent transformation.

Example 3.12:

1. Solve each of the following inequality in the domain of $\mathbb{Q}$.

a) $3x - 7 \geq 11$

b) $\frac{1}{4}x + \frac{3}{2} < \frac{1}{3}$

c) $x + 4 \leq 4x - 3$

d) $-7(x + 1) < 4(x - 3) + 6$

e) $\frac{1}{2}x - 4 < \frac{1}{4}(2x - 5)$

Solution:

a) $3x - 7 \geq 11$

$3x - 7 + 7 \geq 11 + 7$

$3x \geq 18$

$\frac{1}{3} \times 3x \geq \frac{1}{3} \times 18$

$x \geq 6 \quad , x \in \mathbb{Q}$

b) $\frac{1}{4}x + \frac{3}{2} < \frac{1}{3}$

$\frac{1}{4}x + \frac{3}{2} - \frac{3}{2} < \frac{1}{3} - \frac{3}{2}$

$\frac{1}{4}x < -\frac{7}{6}$

$4 \times \frac{1}{4}x < 4 \times -\frac{7}{6}$

$x < -\frac{14}{3}, x \in \mathbb{Q}$

c) $x + 4 \leq 4x - 3$

$x + 4 - 4 \leq 4x - 3 - 4$

$x \leq 4x - 7$

$x + 7 \leq 4x - 7 + 7$

$x + 7 \leq 4x$

$x + 7 - x \leq 4x - x$

$7 \leq 3x$

$\frac{7}{3} \leq \frac{3x}{3}$

$\frac{7}{3} \leq x$

d) $-7(x + 1) < 4(x - 3) + 6$

$-7x - 7 < 4x - 12 + 6$

$-7x - 7 + 7 < 4x - 6 + 7$

$-7x < 4x + 1$

$-7x - 4x < 4x + 1 - 4x$

$-11x < 1$

$\frac{-11}{-11}x > \frac{1}{-11}$

$x > -\frac{1}{11}, x \in \mathbb{Q}$

$x \geq \frac{7}{3}, x \in \mathbb{Q}$

e) $\frac{1}{2}x - 4 < \frac{1}{4}(2x - 5)$

$$\frac{1}{2}x - 4 < \frac{1}{2}x - \frac{5}{4}$$

$$\frac{1}{2}x - 4 + 4 < \frac{1}{2}x - \frac{5}{4} + 4$$

$$\frac{1}{2}x < \frac{1}{2}x + \frac{11}{4}$$

$$\frac{1}{2}x - \frac{1}{2}x < \frac{1}{2}x + \frac{11}{4} - \frac{1}{2}x$$

$0 < \frac{11}{4}$ **which is always true.**

Therefore, any rational numbers satisfy the given inequality.

Note:

- If an inequality, by means of equivalent transformations, leads to an equivalent inequality which is a true statement, then every element of the domain is a solution of the inequality.
- If an inequality, by means of equivalent transformations, leads to an equivalent inequality which is a false statement, then no element of the domain is a solution of the inequality.

For instance: $5x + \frac{3}{4} > 11x + 1 - 6x$ has no solution in the set of rational numbers.

2) Solve the inequality $3x < 9x + 4$ and sketch the solution set on the number line.

Solution:

$$3x < 9x + 4$$

$$3x - 9x < 9x + 4 - 9x$$

$$-6x < 4$$

$$-\frac{1}{6}(-6x) > -\frac{1}{6}(4)$$

$$x > -\frac{2}{3}$$

The solution set consists of all numbers greater than $-\frac{2}{3}$. These numbers are graphed as:

Figure 3.10

Exercise 3.5:

1. Solve each of the following inequality in the domain of $\mathbb{Q}$

   a) $2y - 3 < \frac{1}{2}(7 - y)$ b) $(x - 2) + 4(2x + 1) \geq 4(x - 3)$

   c) $\frac{1}{2}(3x - \frac{2}{3}) + 6 \leq x + 5$

2. Is there any value of x in $\mathbb{Q}$ with the property that:

   a) $2x < 2x - 3$? b) $8x > 2(4x - 3)$?

3.4. Applications of Linear Equations and Inequalities

Competencies: At the end of this sub-topic, students should:

- Apply linear equations and inequalities in real life situation.
- Solve linear equations and inequalities real- life problems.

In this sub-topic, you will apply the knowledge acquired on equations and inequalities. Different word problems that relate to our life can be solved using linear equations and inequalities. In order to solve word problems involving linear equations or inequalities, we need to translate verbal sentences into mathematical statements. That is, to solve word problems involving linear equations or inequalities:

i) Carefully read the problem and assign a variable to the unknown

ii) Interpret the word problem with mathematical statement

iii) Finally, solve the unknown

i) Applications of Linear Equations

Example 3.13:

Translate the following algebraic expressions in different word phrases.

a) x – 5 b) 8x

Solution:

$x - 5$	$8x$
• A number minus five	• A number multiplied by eight
• The difference of a number and five	• The product of a number and eight
• Five subtracted from a number	• Eight times a number
• A number decreased by five	
• Five less than a number	

Example 3.14:

The relationship between the temperature readings in Celsius scale C and Fahrenheit scale F is given by $C = \frac{5}{9}(F - 32)$.

a) Express F in terms of C.

b) Using the above relation of C and F, What interval on the Celsius scale corresponds to the temperature of 50 < F?

Solution:

a) $C = \frac{5}{9}(F - 32)$.

$9C = 9 \times \frac{5}{9}(F - 32)$.

$9C = 5(F - 32)$.

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

b) $F = \frac{9}{5}C + 32$

Since F > 50, $F = \frac{9}{5}C + 32 > 50$

$\frac{9}{5}C > 50 - 32$

$C > 10$

Example 3.15:

Three tractor drivers working on a private farm together ploughed 12.4 hectares of land in a shift. The first driver ploughed half of the second and the second ploughed 2.4 hectares more than the third. Find the amount of hectare ploughed by the second driver.

Solution:

Total land ploughed = 12.4 hectares

Land ploughed by the 1st driver = half of the 2nd

Land ploughed by the 2nd driver = 2.4 hectare more than the 3rd

Let the land ploughed by the 3rd driver be x hectare.

Then,

$x + (x + 2.4) + \frac{1}{2}(x + 2.4) = 12.4$

$2x + 2.4 + \frac{1}{2}x + 1.2 = 12.4$

$\frac{5}{2}x + 3.6 = 12.4$

$\frac{5}{2}x = 8.8$

$x = 3.52$

Therefore, the second driver ploughed 3.52 + 2.4 = 5.92 hectares.

Example 3.16:

Three years ago the sum of the ages of a man and his son was 52 years. Now the man is 18 years older than his son. What is the present age of his son?

Solution: Let M = the man's present age and S = the son's present age.

Then, $(M-3) + (S-3) = 52$. But $M = S + 18$

$(M-3) + (S-3) = 52$

$M + S - 6 = 52$

$M + S = 58$

$S + 18 + S = 58$

$2S = 40$

$S = 20$

Therefore, the present age of his son is 20 years.

Example 3.17:

Samuel can do a certain work in 15 days and Saron can do the same work in 10 days. In how many days do they together to finish the work?

Solution:

Let Samuel and Saron together can finish the work in d days.

Let us make the following table

Name	Number of days for doing the work	Part of the work done in one day	Work done in d days.
Samuel	15	$\frac{1}{15}$	$\frac{d}{15}$
Saron	10	$\frac{1}{10}$	$\frac{d}{10}$

By the equation, $\frac{d}{15} + \frac{d}{10} = 1$

$$d\left(\frac{1}{15} + \frac{1}{10}\right) = 1$$

$$d\left(\frac{2+3}{30}\right) = 1$$

$$d = 6$$

Therefore, they together can finish the work in 6 days.

Exercise 3.6:

1. Write the following sentences in a mathematical symbol.
   a) 5 less than 7 times a number is 0.
   b) The quotient of a number and nine is 2 less than the number.
   c) Multiply a number by 2 and add 4, the result you get will be 3 times the number decreased by 7
2. Solve for the variable M in the equation F = $G\frac{m\,M}{r^2}$.
3. In a class there are 42 students. The number of girls is 1.1 times the number of boys. How many boys and girls are there in the class?
4. The sum of the ages of the mother and her daughter is 68 years. The mother is 22 years older than her daughter. How old is her mother?

ii) Application of Linear Inequalities

Example 3.18:

Translate the following expressions involving inequalities

a) The cost p of a pen is less than two times the cost b of a book.
b) The speed of a car, v, was at least 30 km/h.
c) Multiply a number a by two so that it is not less than three times a number b.
d) Adding 7 to three times a number x exceeds 29

Solution:

a) $p < 2b$ b) $v \geq 30$
c) $2a \geq 3b$ d) $3x + 7 > 29$

Note:

The following mathematical interpretations can be used for the phrase:

- x is at least a ------------ $x \geq a$
- x is not less than a -------- $x \geq a$
- x is at most a ---------------- $x \leq a$
- x is not more than a ---------- $x \leq a$

Example 3.19:

Find two consecutive even positive integers whose sum is at most 10.

Solution:

Let x be the first even integer. Then the second will be x + 2.

$$x + (x + 2) \leq 10$$

$$2x + 2 \leq 10$$

$$2x \leq 8$$

$$x \leq 4$$

Since x is a positive even integer, x = 2 or x = 4.

Therefore, the numbers are 2 and 4, or 4 and 6.

Example 3.20:

Three years ago a father's age exceeded at least four times his son's age. If the father is 47 years old now, then what is the possible present age of the son?

Solution:

Let the son's age be S and father's age be F. Then

$$F - 3 > 4(S - 3)$$

$$F - 3 > 4S - 12$$

$$F > 4S - 9$$

$$47 > 4S - 9$$

$$56 > 4S$$

$$S < 14$$

Example 3.21:

Hanan got a new job and will have to move. Her monthly income will be Birr 13,926. To qualify to rent an apartment; Hanan's monthly income must be at least three times as much as the rent. What is the highest rent Hanan will qualify for?

Solution:

Let x be the amount of rent.

Hanan's monthly income must be at least three times the rent. So it can be written in a linear inequality as $13{,}926 \geq 3x$

$$3x \leq 13{,}926$$

$$\frac{3x}{3} \leq \frac{13{,}926}{3}$$

$$x \leq 4642$$

Therefore, the highest rent amount is Birr 4642.

Example 3.22:

A carnival has two plans for tickets.

Plan A: Birr 15 for entrance fee and Birr 10 for each ride.

Plan B: Birr 10 for entrance fee and Birr 15 for each ride. How many rides would you have to take for plan A to be less expensive than plan B?

Solution:

Let x be the number of rides.

Then cost with plan A = $15 + 10x$

cost with plan B = $10 + 15x$

since, cost with plan A < cost with plan B

$$15 + 10x < 10 + 15x$$

$$15 - 10 < 15x - 10x$$

$$5 < 5x$$

$$1 < x$$

So if you plan to take more than one ride, plan A is less expensive.

Exercise 3.7:

1. Translate the following expressions involving inequalities.
   a) Three times a number decreased by four is at most twenty.
   b) The ratio of a number to seven is less than ten.
   c) The height of a roof, h, was no more than 6m.
   d) The sum of two consecutive integers is smaller than three times the smaller integer.
2. Five times a certain natural number is decreased by two times the number is less than 12. What are the possible values of this number?
3. Imagine that you are taking a ride while on vacation. If the ride service charges Birr 50 to pick you up from the hotel and Birr 10 per km for the trip. What maximum km's you travel if your cost is not more than Birr 1350.

SUMMARY FOR UNIT 3

1. Two mutually perpendicular lines, called axes, form the Cartesian coordinate plane.
2. The two axes divide the given plane into four quadrants called I, II, III, and IV quadrants respectively starting from the positive direction of the x- axis and moving to the counter clock wise.

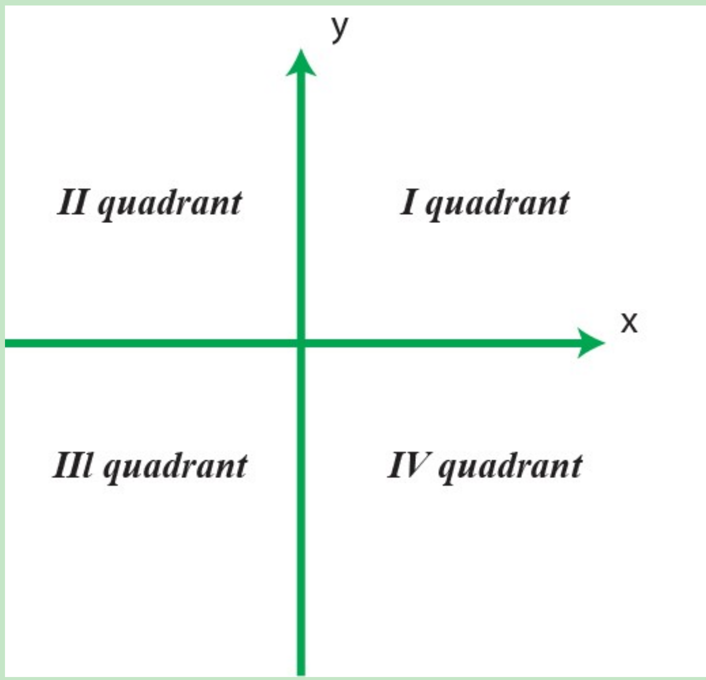


Figure 3.11

3. All ordered pairs that satisfy a linear equation $y = mx$, $m \in \mathbb{Q}, m \neq 0$ is a straight line that passes through the origin. If $m > 0$, the line passes through the I and III quadrant, while if $m < 0$, the line passes through the II and IV quadrant.
4. A mathematical sentence which contains either $<, >, \leq, \geq, or \neq$ is called an inequality.
5. A linear inequality in one variable "x" is an inequality that can be written in the form of " $ax + b < 0$," "$ax + b \leq 0$" or "$ax + b > 0$", or "$ax + b \geq 0$" where a and b are rational numbers and $a \neq 0$.

6. Two linear inequalities are said to be equivalent if and only if they have the same solution set.
7. [Rules of transformation]

   For any rational numbers $a, b,$ and c

   I. If $a < b$, then $a + c < b + c$

   II. If $a < b$, then $a - c < b - c$

   III. If $a < b$ and $c > 0$, then $ac < bc$

   IV) If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$

   V) If $a < b$ and $c < 0$, then $ac > bc$

   VI) If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$

REVIEW EXERCISE FOR UNIT 3

1. Write true for correct statement and false for the incorrect one.
   a) The graph of the line $y = -2x$ passes through the II and III quadrants.
   b) The graph of the equation $x = a,\ a \in \mathbb{Q}$, and $a > 0$ is a vertical line that lies to the right of the y-axis.
   c) A horizontal line has the equation $x - b = 0$.
   d) If $a < b$ and $c = -3$, then $ac > b\,c$
   e) The inequality $8 \geq 5x,\ x \in$ W has a finite solution set.
2. Plot the following points in a Cartesian coordinate plane:
   $(0,\ 6)$, $(2,\ -3)$, $(-4,\ 5)$ and $(-4,\ -5)$. Which point lies in neither of the quadrant?
3. Refer Figure 3.12 and answer the following questions.

a) Name the coordinates of the point D, E and F.
b) Which point has the coordinates $(-2, 1)$?
c) Which coordinate of B is 3?
d) In which quadrant does the point C, E lie?

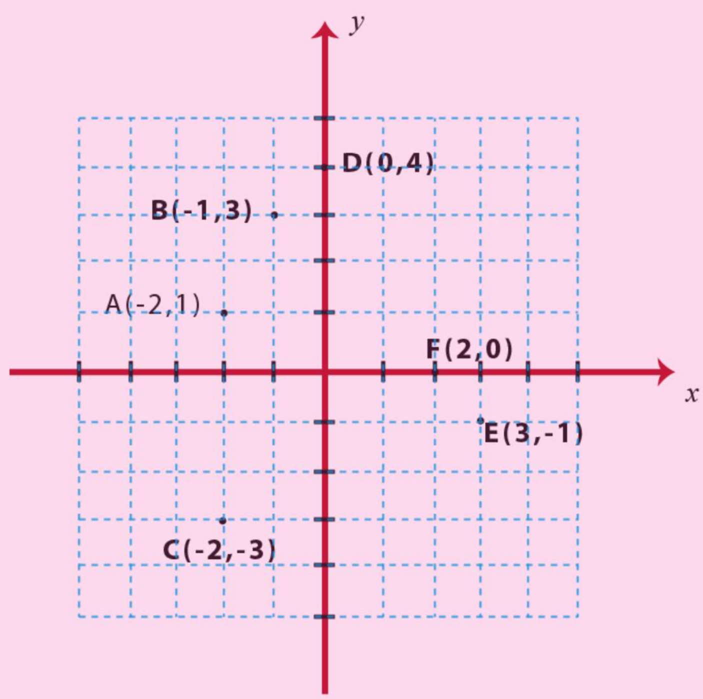


Figure 3.12

4. Fill in the blank with an appropriate inequality sign.

   a) If $x \geq 3$, then $-4x$ ------- -12

   b) If $x + 4 < 2x$, then $\frac{1}{2}x$ ------ 2

   c) If $x < -2$, then $1 - x$ ------ 3

5. Determine whether the given points are on the graph of the equation

   $x - 2y - 1 = 0$

   a) (0, 0) b) (1, 0) c) $(-1, -1)$ d) (2, 1)

6. Sketch the graph of the following equations.

   a) $y = -7x$ b) $y - 11x = 0$
   c) $3y = -7x + 6$ d) $y - 7 = 0$

7. Find a and b, if the points P(3, 1) and Q(0,2) lie on the graph of

   $ax - by = 6$

8. Consider the equations $y = x + 1$ and $y = 1 - x$.

   a) Determine the values of y for each equation when the values of x are -1, 0 and 1.

   b) Plot the ordered pairs on the Cartesian coordinate plane.

   c) Which point is a common point, called intersection point?

9. Solve the following inequalities and graph the solution set.

   a) $3x + 11 \leq 6x + 8$ c) $4 - 3x \leq -\frac{1}{2}(2 + 8x)$

   b) $6 - 2x > x + 9$

10. Solve the following inequalities

    a) $\frac{x}{7} \geq \frac{3}{14}x - \frac{1}{7}$

    b) $2(\frac{1}{2} - x) < 3(1 + \frac{1}{2}x) + 5$

    c) $\frac{1}{2}x + \frac{1}{3}x - x - 1 \geq 0$

    d) $\frac{3}{4}y + \frac{1}{2}(y - 3) < \frac{y + 1}{4}$

    e) $7(y + 1) - y > 2(3y + 4)$

    f) $-4(y - 1) + 3y \geq 1 - y$

11. Solve the following inequalities in the given domain.

a) $\frac{2}{3}x < 4(4-x),\ x \in \mathbb{Z}^{+}$

b) $\frac{1}{6} - \frac{1}{4}x \geq 2 + \frac{2}{3}x,\ x \in \mathrm{W}$

c) $2(3y-7) - 14 \geq 3(2y-11),\ x \in \mathbb{Q}$

12. Translate the following sentences in to mathematical expressions.

a) A year ago a father's age exceeded three times his son's age.

b) Three –fourth of a number is greater than 12.

c) The average mark of Saron is not smaller than 89.

13. A board with 2.5 m in length must be cut so that one piece is 30 cm more than the other piece. Find the length of each piece.

Figure 3.13

14. Rodas is 25 years old and her brother Mathanya is 10 years old. After how many years will Rodas be exactly twice as old as Mathanya.

15. Yoseph wants to surprise his wife with a birthday party at her favourite restaurant. It will cost Birr 56.50 per person for dinner, including tip and tax. His budget for the party is Birr 735. What is the maximum number of people Yoseph can have at the party?

16. Getaneh and Salhedin play in the same soccer team. Last Saturday Salhedin scored 3 more goals than Getaneh, but together they scored less than 7 goals. What are the possible number of goals Salhedin scored?

UNIT 4

SIMILARITY OF FIGURES

Learning Outcomes: At the end of this unit, learners will able to:

- Know the concept of similar figures and related terminologies.
- Understand the condition for triangles being similar.
- Apply tests to check whether two given triangles are similar or not.
- Apply real-life situations in solving geometric problems.

Main Contents

Main Contents

4.1 Similar plane figures

4.2 Perimeter and Area of Similar Triangles

Summary

Review Exercise

4.1. Similar Plane Figures

Competencies: At the end of this sub-topic, students should:

- Identify figures that are similar to each other.
- Apply the definition of similarity of two triangles to solve related problems.
- Determine the similarity of two triangles.

INTRODUCTION

This unit focused on similarity of plane figures. In common language the word similar can have many meanings but in mathematics, the word similar and similarity have very specific meanings. In mathematics, we say that two objects are similar if they have the same shape, but are not necessarily the same size. For instance, an overhead projector forms an image on the screen which has the same shape as the image on the transparency but with the size altered. Two figures that have the same shape but not necessarily the same size are called similar figures.

Activity 4.1

1. Which of the following figures are similar?

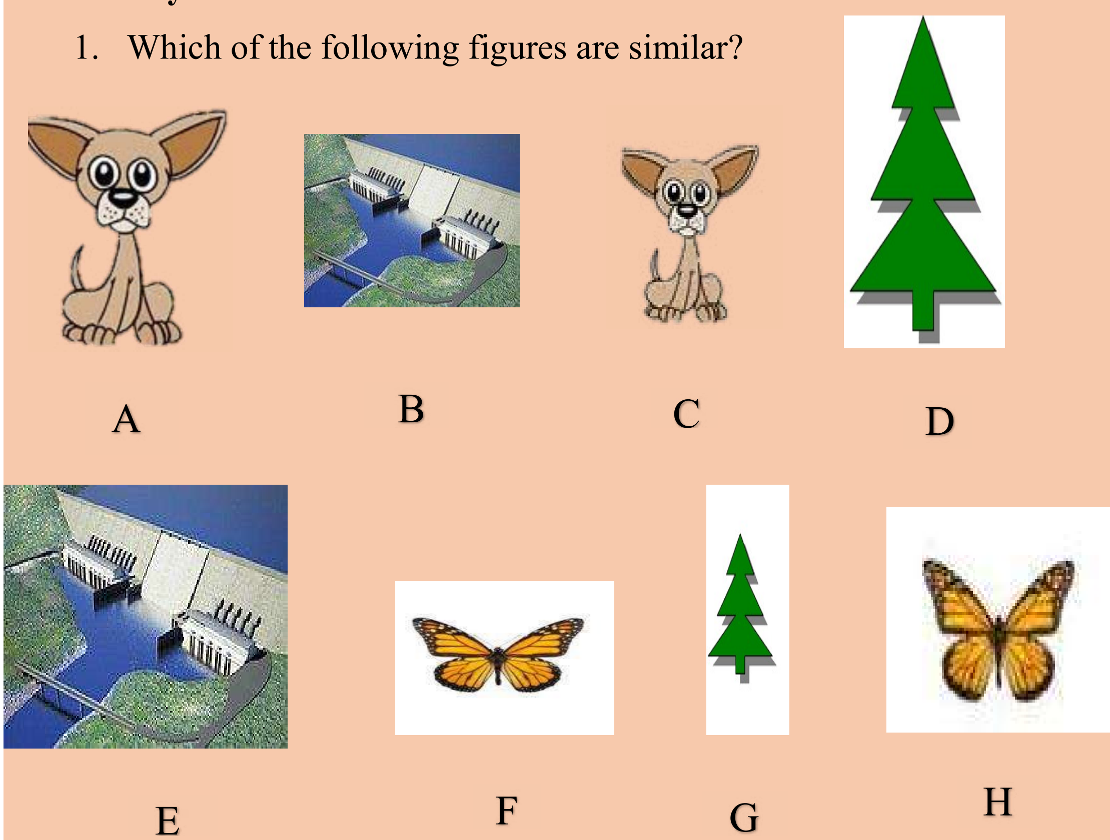


Figure 4.1

2. Are any two equilateral triangles always congruent?
3. Let ABCD be a square and $\overline{AC}$ be its diagonal. Are Δ ABC $\cong$ Δ ADC?
4. In the triangles, Δ ABC and Δ PQR, if $\angle A \cong \angle P$, and $\angle B \cong \angle Q$, then what is the relation between each $\angle C$ and $\angle R$?
5. Decide whether the polygons are similar.
   a. Rectangle and parallelogram
   b. Square and trapezium

4.1.1. Definition and Illustration of Similar Figures

In this section, we will discuss how to compare the size and shape of two given figures.

Recall that, one way to determine whether two geometric figures are congruent is to see if one figure can be moved on to the other figure in such a way that it "fits exactly".

If two figures have the same shape, but not necessarily the same size (That is, one figure is an exact scale model of the other figure), then we say that the two geometric figures are similar. The symbol $\sim$ (read as "is similar to") is used for the term similar.

Example 4.1:

A tree and its shadow are similar.

Example 4.2:

The two photograph of the same size of the same person, one at the age of 5 years and the other at the age of 50 years are not similar.

Because similar figures differ only in size, there is a test we can perform to make sure that our shapes are really similar.

Similar Polygons

Definition 4.1: Two polygons are said to be similar if there is a one – to – one correspondence between their vertices such that:

i) all pairs of corresponding angles are congruent

ii) the ratio of the lengths of all pairs of corresponding sides are equal.

In the diagram, ABCD is similar to EFGH.

That is, ABCD ~ EFGH, then

i) $\angle A \cong \angle E, \quad \angle B \cong \angle F, \quad \angle C \cong \angle G, \quad \angle D \cong \angle H$

ii) $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$

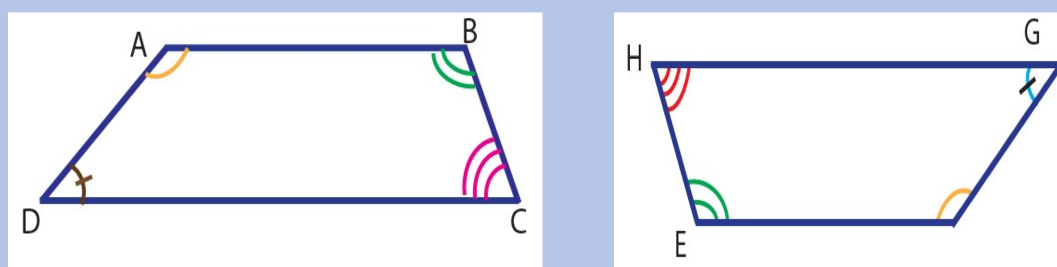


Figure 4.2

Example 4.3:

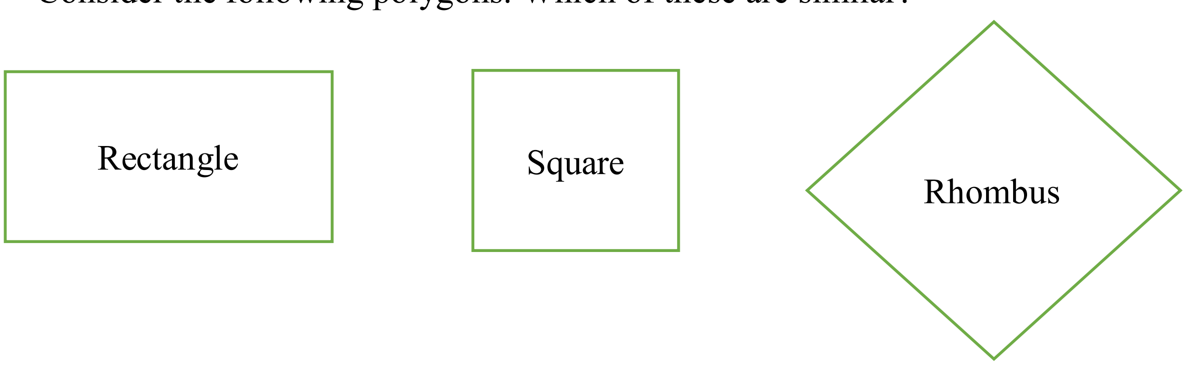
The following pairs of figures are always similar

a) Any two squares.

b) Any two equilateral triangles.

Example 4.4:

Consider the following polygons. Which of these are similar?



Solution:

The square and the rectangle are not similar. Because their corresponding angles are congruent but their corresponding sides are not proportional. The square and the rhombus are not similar. Because their corresponding sides are proportional but their corresponding angles are not congruent. The rectangle and the rhombus are not similar. Because their corresponding angles are not congruent and corresponding sides are not proportional.

Definition: 4.2. The ratio of two corresponding sides of similar polygons is called the scale factor or constant of proportionality (k).

Example 4.5:

The sides of a quadrilateral are 2cm, 5cm, 6cm and 8cm. Find the sides of a similar quadrilateral whose shortest side is 3cm.

Solution:

Let the corresponding sides of a quadrilateral are 3, x, y, and z. Since the corresponding sides of similar polygons are proportional,

$$\frac{2}{3} = \frac{5}{x}, \quad \frac{2}{3} = \frac{6}{y}, \qquad \frac{2}{3} = \frac{8}{z}$$

$$x = \frac{15}{2}, \qquad y = 9, \qquad z = 12$$

Therefore, the sides of the second quadrilateral are 3cm, 7.5cm, 9cm, and 12cm.

Example 4.6:

Decide whether the polygons are similar. If so find the scale factor of Figure A to Figure B.

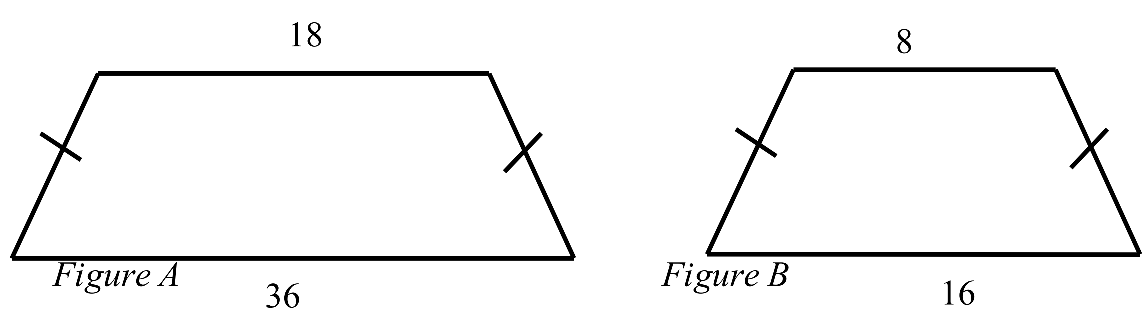


Figure 4.3